FEEDBACK IN AMPLIFIERS*

5-1. Principles of Feedback.[1] When a part of the output signal is combined with the input signal, feedback is said to exist. If the net effect of the feedback is to increase the effective input signal, the feedback, is called *positive*, *direct*, or *regenerative*. If the resultant input signal is reduced by the feedback potential, the feedback is called *negative*, *inverse*, or *degenerative*.

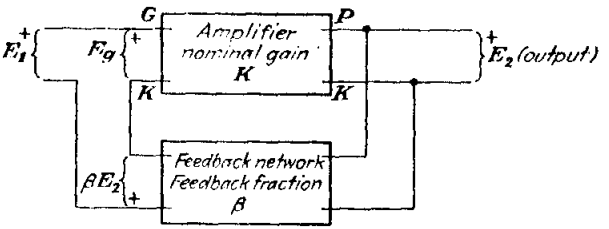


FIG. 5-1. The principle of feedback in amplifiers.

The principle of feedback is illustrated in the schematic diagram of Fig. 5-1. For simplicity, series injection is shown at the input, but other forms of network coupling may be employed. In the diagram shown, a potential $\mathbf{E}_1$ is applied to the input terminals of the amplifier. Suppose that the resultant potential at the output terminals is $\mathbf{E}_2$. Now suppose that a fraction β of this output is fed back in series with the input signal in such a way that the resultant signal that appears between the grid and cathode terminals has the form

$$\mathbf{E}_g = \mathbf{E}_1 + \beta\mathbf{E}_2 \tag{5-1}$$

But the *nominal gain* of the amplifier is given by

$$\mathbf{K} \equiv \frac{\text{output potential}}{\text{potential between grid and cathode}} = \frac{\mathbf{E}_2}{\mathbf{E}_g}$$

Then

$$\mathbf{E}_2 = \mathbf{K}\mathbf{E}_g \tag{5-2}$$

* Part of the contents of this chapter was originally prepared with Dr. J. Millman for the second edition of "Electronics," McGraw-Hill Book Company, Inc., New York, 1951, although the material was not included in that text.

Observe that the nominal gain requires the injection of a potential $\mathbf{E}_g$ between the grid-cathode terminals, with an evaluation of the output potential $\mathbf{E}_2$, with the β network acting as part of the total output load of the circuit.

Equation (5-2) is combined with Eq. (5-1) to yield

$$\mathbf{E}_2 = \mathbf{K}\mathbf{E}_1 + \mathbf{K}\beta\mathbf{E}_2$$

from which it follows that

$$\mathbf{E}_2 = \frac{\mathbf{K}\mathbf{E}_1}{1 - \mathbf{K}\beta} \tag{5-3}$$

The *resultant gain* of the amplifier with feedback is defined as

$$\mathbf{K}_f \equiv \frac{\text{output potential}}{\text{input signal potential}} = \frac{\mathbf{E}_2}{\mathbf{E}_1}$$

Therefore it follows that

$$\mathbf{K}_f = \frac{\mathbf{K}}{1 - \mathbf{K}\beta} \tag{5-4}$$

This equation expresses the resultant gain of the amplifier with feedback $\mathbf{K}_f$ in terms of the nominal gain of the amplifier without feedback $\mathbf{K}$, and the feedback fraction β.

It can be seen that if $|1 - \mathbf{K}\beta|$ is greater than unity, then $\mathbf{K}_f$ is less than $\mathbf{K}$. The feedback is then said to be negative, or degenerative. The application of negative feedback to an amplifier results in a number of characteristics that are highly desirable in the amplifier. It tends to flatten the frequency-response characteristic and to extend the range of uniform response. It materially reduces nonlinear and phase distortion. It improves the stability of the amplifier, making the gain less dependent on the operating potentials or on variations of the tube characteristics. Also, it tends to make the gain less dependent on the load, so that load variations do not seriously influence the operating characteristics of the amplifier. The use of feedback networks of special design will provide selective attenuation, thus permitting a frequency response of desired characteristic. A detailed discussion of the foregoing features will be given later.

Conversely, if $|1 - \mathbf{K}\beta|$ is less than unity, then $\mathbf{K}_f$ is greater than $\mathbf{K}$. The feedback is now termed positive, or regenerative. The application of positive feedback has effects opposite to those with negative feedback. Thus positive feedback tends to sharpen the frequency-response curve and to decrease the range of uniform response. This permits an increased gain and selectivity. Positive feedback in any amplifier is critical of adjustment. Too much regenerative feedback in any system may result in oscillation. Ordinarily, negative feedback is more common than positive feedback in amplifiers, although oscillators of the feedback variety depend for their operation on the presence of positive feedback.

Observe that, for the case when $\mathbf{K}\beta = 1 + j0$, the gain becomes infinite. In this case the amplifier becomes an oscillator, and the output potential is independent of any external signal potential.

Attention is called to the fact that the action of a feedback path depends upon the frequency of operation. That is, the feedback may remain regenerative or degenerative throughout the range of operation of the circuit, although the magnitude and phase angle of the feedback signal may vary with the frequency. It is also possible for the feedback to be positive over a certain range of frequencies and negative over another range of frequencies.

Example. The circuit of a simple triode amplifier with an impedance in the cathode lead is illustrated in Fig. 5-2. This circuit is to be analyzed by two

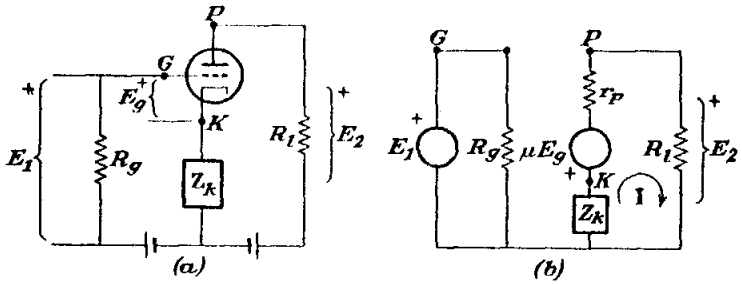


FIG. 5-2. A simple amplifier with cathode degeneration.

methods. One method is a direct application of the feedback equation [Eq. (5-4)]. The second method is a direct application of electron-tube circuit principles.

Solution. Refer to the equivalent circuit of the amplifier which is given in Fig. 5-2*b*. Observe that a part of the output is fed back into the input circuit through the impedance $\mathbf{Z}_k$. It follows from the figure that

$$\mathbf{E}_g = \mathbf{E}_1 + \mathbf{I}\mathbf{Z}_k \tag{5-5}$$

But since

$$\mathbf{I} = \frac{\mathbf{E}_2}{R_l} \tag{5-6}$$

then

$$\mathbf{E}_g = \mathbf{E}_1 + \frac{\mathbf{Z}_k}{R_l}\mathbf{E}_2 \tag{5-7}$$

If this expression is compared with Eq. (5-1), which defines the feedback fraction, it is seen that

$$\beta = \frac{\mathbf{Z}_k}{R_l} \tag{5-8}$$

Note also that

$$\mathbf{E}_2 = \mathbf{I}R_l = -\frac{\mu\mathbf{E}_g}{r_p + \mathbf{Z}_k + R_l}R_l \tag{5-9}$$

and the nominal gain then becomes

$$\mathbf{K} = \frac{\mathbf{E}_2}{\mathbf{E}_g} = \frac{-\mu R_l}{r_p + \mathbf{Z}_k + R_l} \tag{5-10}$$

The resultant gain is, by Eq. (5-4),

$$K_f = \frac{\mathbf{K}}{1 - \mathbf{K}\beta} = \frac{-\mu R_l/(r_p + \mathbf{Z}_k + R_l)}{1 - \dfrac{-\mu R_l}{r_p + \mathbf{Z}_k + R_l}\dfrac{\mathbf{Z}_k}{R_l}} = \frac{-\mu R_l}{r_p + (\mu + 1)\mathbf{Z}_k + R_l} \tag{5-11}$$

These results follow of course from direct considerations of the equivalent circuit. The plate circuit yields the expression

$$\mu \mathbf{E}_g + \mathbf{I}(r_p + \mathbf{Z}_k + R_l) = 0 \tag{5-12}$$

But

$$\mathbf{E}_g = \mathbf{E}_1 + \mathbf{I}\mathbf{Z}_k \tag{5-13}$$

The solution of these equations gives

$$\mu(\mathbf{E}_1 + \mathbf{I}\mathbf{Z}_k) + \mathbf{I}(r_p + \mathbf{Z}_k + R_l) = 0$$

from which

$$\mathbf{I} = \frac{-\mu \mathbf{E}_1}{r_p + (\mu + 1)\mathbf{Z}_k + R_l} \tag{5-14}$$

and the resultant gain is

$$\mathbf{K}_f = \frac{\mathbf{E}_2}{\mathbf{E}_1} = \frac{-\mu R_l}{r_p + (\mu + 1)\mathbf{Z}_k + R_l} \tag{5-15}$$

which is the same as above.

Often $\mathbf{Z}_k$ consists of the parallel combination of R_k and C_k, the value of R_k being so chosen that $I_b R_k$ is just equal to E_c, the quiescent d-c bias of the tube. The capacitance C_k is so chosen that its reactance is very small over the operating range of the amplifier. As a result, $\mathbf{Z}_k$ is very small and may be omitted in the above expression. In this case, the usual simple amplifier formula is obtained, since the feedback factor β is zero, and no feedback exists.

5-2. Feedback Amplifier Characteristics. The presence of negative feedback in an amplifier results in a number of desirable characteristics. These are discussed below.

1. *Stability of Amplification.* Suppose that the feedback is negative and that the feedback factor $\mathbf{K}\beta$ is made large compared with unity. The resultant gain equation (5-4) becomes

$$\mathbf{K}_f \doteq -\frac{\mathbf{K}}{\mathbf{K}\beta} = -\frac{1}{\beta} \tag{5-16}$$

This means that when the magnitude $K\beta \gg 1$, the actual amplification with negative feedback is a function of the characteristics of the feedback network only. In particular, if β is independent of frequency, then the over-all gain will be independent of the frequency. This permits a substantial reduction of the frequency and phase distortion of the amplifier. In fact, by the proper choice of feedback network, it is possible to achieve almost any desired frequency characteristic.

Note that if $\mathbf{K}\beta \gg 1$, then $\mathbf{K}_f \doteq -\mathbf{K}/\mathbf{K}\beta \ll -\mathbf{K}$, so that the over-all gain of the amplifier with inverse feedback is less than the nominal gain without feedback. This is the price that must be paid to secure the advantages of negative feedback. This is not a serious price to pay, since the loss in gain can be overcome by the use of additional tubes.

Clearly, if $\mathbf{K}\beta$ is greater than unity, then Eq. (5-16) shows that the over-all gain will not change with tube replacements or with variations in battery potentials, since β is independent of the tube. Even if Eq. (5-16) is not completely valid, a substantial improvement results in general stability. This follows from the fact that a change in the nominal gain $d\mathbf{K}$ for whatever reason results in a change $d\mathbf{K}_f$ in the resultant gain by an amount

$$\frac{d\mathbf{K}_f}{\mathbf{K}_f} = \frac{1}{|1 - \mathbf{K}\beta|}\frac{d\mathbf{K}}{\mathbf{K}} \tag{5-17}$$

where $|1 - \mathbf{K}\beta|$ represents the magnitude of the quantity $1 - \mathbf{K}\beta$. This equation is the logarithmic derivative of Eq. (5-4). In this expression, $d\mathbf{K}_f/\mathbf{K}_f$ gives the fractional change in $\mathbf{K}_f$, and $d\mathbf{K}/\mathbf{K}$ gives the fractional change in $\mathbf{K}$. If, for example, the quantity $|1 - \mathbf{K}\beta| = 5$ in a particular feedback amplifier, then the variation in any parameter that might cause a 5 per cent change in the nominal gain will result in a change of only 1 per cent in the resultant gain of the amplifier.

2. *Reduction of Frequency and Phase Distortion.* It follows from Eqs. (5-4) and (5-16) that the over-all gain of the amplifier is almost independent of frequency, provided that β is frequency-independent. In such cases the frequency and phase distortion of an amplifier are materially reduced below the nonfeedback value.

3. *Reduction of Nonlinear Distortion.* One effect was omitted in the above considerations. It was implicitly assumed that the dynamic curve was linear and that the output potential was of the same waveshape as the input. If an appreciable nonlinear distortion exists, then the output contains harmonic components in addition to the signal of fundamental frequency. Suppose, for simplicity, that only a second-harmonic component B_2 is generated within the tube when a large signal potential is impressed on the input. Because of the feedback, the second-harmonic component B_2' that appears in the output is different from that generated within the tube. To find the relationship that exists between B_2' and B_2, the procedure parallels that for the gain considerations. Thus, for a second harmonic B_2' in the output, a fraction $\beta B_2'$ is supplied to the input. As a result, the output actually must contain two components of second-harmonic frequency, the component B_2 that is generated within the tube

and the component $\mathbf{K}\beta B_2'$ that arises from the signal that is fed back to the input. This requires that

$$\mathbf{K}\beta B_2' + B_2 = B_2'$$

or

$$B_2' = \frac{B_2}{1 - \mathbf{K}\beta} \tag{5-18}$$

Note that since both $\mathbf{K}$ and β are functions of the frequency, in general, the appropriate values that appear in this equation must be evaluated at the second-harmonic frequency.

It should be pointed out that this derivation has assumed that the harmonic distortion generated within the tube depends only upon the grid swing of the fundamental signal potential. The small amount of additional distortion that might arise because a fraction of the second-harmonic component is returned to the input has been neglected. Ordinarily this procedure will lead to little error, although a more exact calculation taking these successive effects into account is readily possible.[2]

Another feature of Eq. (5-18) should be noted. According to this expression, if $|1 - \mathbf{K}\beta| = 10$, then the second-harmonic distortion with feedback is only one-tenth its value without feedback. This is the situation when the total output-potential swing is the same in each case; otherwise the harmonic generation within the tube could not be directly compared. This requires that the signal, when feedback is applied, must be $|1 - \mathbf{K}\beta|$ times that in the absence of feedback. As a practical consideration, since appreciable nonlinear distortion is generated only when the signal potential is large, then the full benefit of the feedback amplifier in reducing nonlinear distortion is obtained by applying negative feedback to the large-signal stages.

4. *Reduction of Noise.* Considerations such as those leading to Eq. (5-18) for the resultant nonlinear distortion in a feedback amplifier will show that the resultant noise in an amplifier is reduced by the factor $1 - \mathbf{K}\beta$, when feedback is employed. This would seem to represent a real reduction in noise. However, if the requirement is for a specified output signal, the resultant gain with feedback will have to be adjusted, by adjustment of the circuit parameters, or by the addition of amplifier stages, to give the same over-all gain as the amplifier without feedback. Consequently, the noise will be amplified as well as the signal. Moreover, since the noise is independent of the signal, additional amplifier stages to compensate for the loss of gain due to feedback will introduce additional noise. In such cases, the over-all noise of the amplifier with feedback might be higher than that of one without feedback. If the required gain is achieved by the readjustment of the circuit parameters, a reduction in noise will result in the negative feedback amplifier.

5. *Modification of Input and Output (Effective Internal) Impedances.* These topics will be the subject of detailed consideration in several of the following sections.

5-3. Feedback Circuits. The potential fed back from the output of the amplifier into the input may be proportional either to the potential across the load or to the current through the load. In the first case, the feedback is called *potential feedback;* in the second case, it is called *current feedback.* In either case, the feedback may be positive or negative, depending upon the connection. Often the feedback loops are so involved and interconnected that it is not possible to specify directly

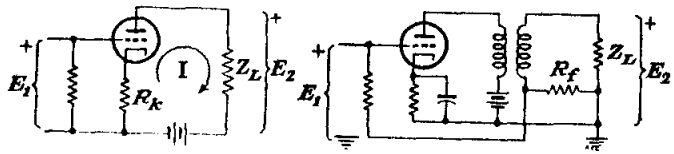


FIG. 5-3. Circuits employing current feedback.

whether the feedback is of the potential or the current types or whether a combination of both exists.

It is possible to state rules which help to specify more uniquely the existence of potential or current feedback. Consider the circuits of Fig. 5-3, which illustrate two amplifiers employing current feedback. The first of these diagrams is identical with that of the illustrative example of the foregoing section, except that the cathode impedance is now shown as a resistance R_k. As in the example, the feedback ratio is $\beta = R_k/\mathbf{Z}_l$. Note that, for large feedback ratios, the resultant gain approaches

$$\mathbf{K}_f = -\frac{1}{\beta} = -\frac{\mathbf{Z}_l}{R_k} \tag{5-19}$$

Therefore the output potential is

$$\mathbf{E}_2 = -\frac{\mathbf{Z}_l}{R_k}\mathbf{E}_1 \tag{5-20}$$

which is proportional to the load impedance. Also, the output current is given by

$$\mathbf{I} = \frac{\mathbf{E}_2}{\mathbf{Z}_l} = -\frac{\mathbf{E}_1}{R_k} \tag{5-21}$$

which is seen to be independent of the load impedance. These conditions are characteristic of current feedback. Hence in current feedback *the ratio of the feedback potential to the load current is independent of the load impedance.*

The condition that the output current should be independent of the load impedance is fulfilled when the internal impedance of the generator is high compared with the impedance of the external load. Conse-

quently, current feedback has the property of increasing the internal impedance of the network. In fact, from the complete expression for the current, Eq. (5-14), namely,

$$\mathbf{I} = \frac{-\mu \mathbf{E}_1}{r_p + (\mu + 1)R_k + \mathbf{Z}_l}$$

it is possible to draw Fig. 5-4, which is the equivalent of Fig. 5-2*a*. It follows from this that the circuit including feedback comprises a potential source $\mathbf{E}_{tf} = \mu \mathbf{E}_1$ with an internal impedance $\mathbf{Z}_{tf} = r_p + (\mu + 1)R_k$. Since the internal impedance without feedback is simply $r_p + R_k$, the effect of the feedback is to increase the internal impedance by the term μR_k. The ratio of internal impedances with and without feedback is given by

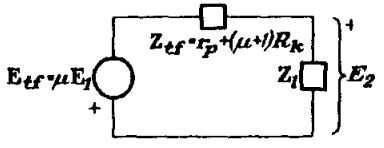


Fig. 5-4. The equivalent circuit of Fig. 5-2*a*.

$$\frac{\mathbf{Z}_{tf}}{\mathbf{Z}_l} = \frac{r_p + (\mu + 1)R_k}{r_p + R_k} = 1 + \frac{\mu R_k}{r_p + R_k} \tag{5-22}$$

A circuit which employs potential feedback is given in Fig. 5-5. In this circuit, the resistance combination $\beta R + (1 - \beta)R = R$ which shunts the output is made large compared with the load impedance $\mathbf{Z}_l$. The capacitor C has a reactance that is negligible compared with R at the frequencies to be employed. Its sole purpose is to block the d-c potential from the plate circuit from appearing in the grid circuit.

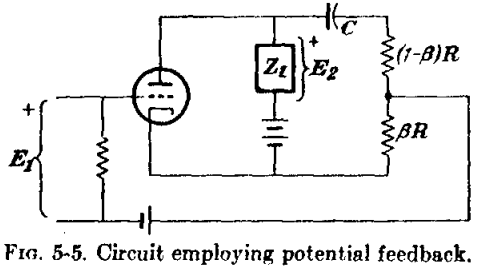


Fig. 5-5. Circuit employing potential feedback.

The feedback ratio is shown as β in the diagram. Also for large feedback ratios, the resultant gain approaches

$$\mathbf{K}_f = -\frac{1}{\beta}$$

The output potential is, therefore,

$$\mathbf{E}_2 = -\frac{1}{\beta}\mathbf{E}_1$$

which is seen to be independent of the load impedance, since β is independent of $\mathbf{Z}_l$. Observe therefore that potential feedback is directly proportional to the output potential, and the *ratio β of the feedback potential to the output potential is independent of the load.* A generator whose output potential is substantially independent of the load impedance must possess a very low internal impedance. Consequently potential feedback has the property of decreasing the internal impedance of the amplifier.

To obtain an expression for the resultant gain of the amplifier, the feedback method will be employed. By neglecting the shunting effect of the feedback resistance network on the load impedance, it follows that the nominal gain of the amplifier is given by

$$\mathbf{K} = \frac{-\mu \mathbf{Z}_l}{r_p + \mathbf{Z}_l} \tag{5-23}$$

Then the resultant gain with feedback is

$$\mathbf{K}_f = \frac{\mathbf{K}}{1 - \mathbf{K}\beta} = \frac{-\mu \mathbf{Z}_l}{r_p + \mathbf{Z}_l + \beta\mu \mathbf{Z}_l} \tag{5-24}$$

This expression may be transformed to the form

$$\mathbf{K}_f = \frac{-\mu' \mathbf{Z}_l}{r'_p + \mathbf{Z}_l} \tag{5-25}$$

where $\mu' = \dfrac{\mu}{1 + \mu\beta}$ $\qquad r'_p = \dfrac{r_p}{1 + \mu\beta}$

But this is exactly the output that is obtained from the circuit of Fig. 5-6. Consequently, the circuit behaves like a potential source $\mathbf{E}_{tf} = \dfrac{\mu}{1 + \mu\beta}\mathbf{E}_1$ with an internal impedance $\mathbf{Z}_{tf} = r_p/(1 + \mu\beta)$

The effective internal impedance of the amplifier without feedback is simply $\mathbf{Z}_t = r_p$. The effect of potential feedback is to reduce the internal impedance in the ratio

$$\frac{\mathbf{Z}_{tf}}{\mathbf{Z}_t} = \frac{1}{1 + \mu\beta} \tag{5-26}$$

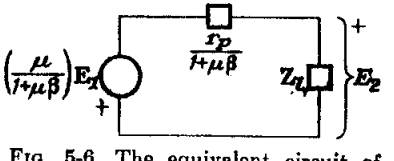


Fig. 5-6. The equivalent circuit of Fig. 5-5.

From the form of Eq. (5-25), the circuit gain appears to be that obtained from a tube whose amplification factor is μ' and whose plate resistance is r'_p. Note that the effective amplification factor is reduced in the same ratio as the plate resistance of the tube. This indicates that a tube possessing a high plate resistance can be effectively converted into a low-plate-resistance tube and thereby permit an impedance match to a low impedance load. This is accomplished, of course, at the expense of effectively converting the tube into a triode, with low μ and low r_p.

The combination of current and potential feedback in an amplifier is frequently called compound, or bridge, feedback. The circuit of such an amplifier is given in Fig. 5-7. The feedback fraction is found to be

$$\beta = \beta_1 + \frac{R_k}{Z_l} = \beta_1 + \beta_2 \tag{5-27}$$

As in the analysis of Fig. 5-5, it is assumed that the resistance combination R is much greater than Z_l and that the reactance of the capacitor is negligible over the frequency range of operation. The resultant gain of the amplifier has the form

$$K_f = \frac{-\mu Z_l}{r_p + (\mu + 1)R_k + (1 + \mu\beta_1)Z_l}$$

This may be written in the form

$$K_f'' = \frac{-\mu'' Z_l}{r_p'' + Z_l}$$

where

$$\mu'' = \frac{\mu}{1 + \mu\beta_1} \tag{5-28}$$

$$r_p'' = \frac{r_p + (\mu + 1)R_k}{1 + \mu\beta_1}$$

The corresponding equivalent circuit shown in Fig. 5-8 gives rise to exactly this expression for the gain and is therefore the equivalent of

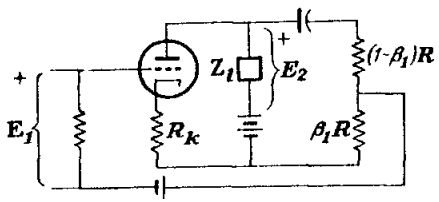


FIG. 5-7. A circuit employing bridge, or compound, feedback.

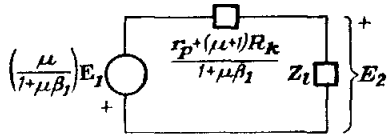


FIG. 5-8. The equivalent network of Fig. 5-7.

Fig. 5-7. The effect of the feedback is seen to reflect itself as a change in the effective μ and r_p of the tube. The effective potential and internal impedance are given by the expression

$$E_{tf} = \frac{\mu}{1 + \mu\beta_1} E_1$$

$$Z_{tf} = \frac{r_p + (\mu + 1)R_k}{1 + \mu\beta_1} \tag{5-29}$$

Owing to the form of the expression for Z_{tf}, this quantity may be made greater than, equal to, or less than its value without feedback.

Feedback can be effected over several stages and need not be limited to a stage-by-stage practice. A two-stage RC-coupled amplifier which combines current feedback in the first stage through resistor R_1 and potential feedback between stages is illustrated in Fig. 5-9. A careful consideration of the polarity of the potentials which are fed back will show that both types of feedback are negative.

It is not always evident what type of feedback is being employed in a given amplifier. The following tests will serve to clarify the situation:

1. If the ratio of feedback potential E_f to output potential E_2 is independent of the load impedance, then potential feedback is employed. This ratio is the feedback fraction $\beta = E_f/E_2$.

2. If the ratio of feedback potential E_f to load current I is independent of the load impedance, then current feedback is employed. The ratio $Z_f = E_f/I$ is the feedback impedance.

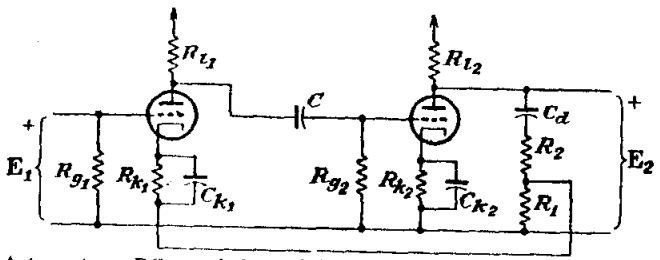


FIG. 5-9. A two-stage RC coupled amplifier with current feedback in the first stage and potential feedback between stages.

3. If the feedback potential E_f is the sum of two terms of the form

$$E_f = \beta E_2 + Z_f I$$

where *both* β and Z_f are independent of the load impedance, then compound feedback is employed.

5-4. Effective Internal Impedance with Feedback. The discussion in the foregoing section has shown that the effective internal impedance of the equivalent plate circuit of an electron-tube circuit with feedback depends on the type of feedback that is employed. As shown, current feedback increases the effective internal impedance, and potential feedback decreases the effective internal impedance. These results will be generalized.

The following notation, some of which has already appeared, will apply in the following development:

β is feedback ratio
K is potential gain without feedback, with load connected
K_f is potential gain with feedback, with load connected
E_t is potential gain without feedback, with load open-circuited
E_t is effective internal potential source without feedback (this is the Helmholtz-Thévenin potential source obtained on open circuit)

$\mathbf{E}_{tf}$ is effective internal potential source with feedback
$\mathbf{Z}_1$ is input-terminal impedance without feedback
$\mathbf{Z}_{1f}$ is corresponding input-terminal impedance with feedback
$\mathbf{Z}_t$ is effective internal impedance without feedback (this is the Thévenin impedance of the equivalent network and is the impedance looking back into the output terminals of the amplifier, with the load open-circuited)
$\mathbf{Z}_{tf}$ is corresponding effective internal impedance with feedback
$\mathbf{Z}_0$ is output-terminal impedance without feedback
$\mathbf{Z}_{0f}$ is output-terminal impedance with feedback
$\mathbf{E}_2$ is output potential
$\mathbf{E}_1$ is input potential to amplifier
$\mathbf{E}_f$ is feedback potential

(Refer to Appendix A for a general discussion of the Helmholtz-Thévenin theorem.) Refer to Fig. 5-10, which shows a general feedback network

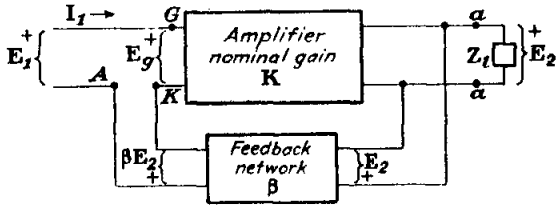


FIG. 5-10. The general potential or parallel feedback circuit.

which is provided with potential feedback. An expression for the internal impedance $\mathbf{Z}_{tf}$ of this feedback network will be derived in terms of the internal impedance $\mathbf{Z}_t$ without feedback.

Consider first the amplifier with the feedback potential removed. This is accomplished by removing lead A from the feedback network and connecting it to the cathode K. The Thévenin potential-source equivalent of this circuit is given in Fig. 5-11. $\mathbf{Z}_t$ in this diagram is the effective internal impedance without feedback, and $\mathbf{K}_t$ is the gain without feedback on open circuit (with $\mathbf{Z}_t$ omitted from the diagram).

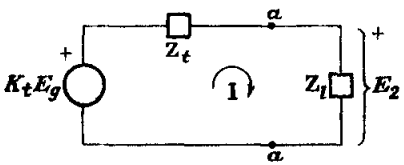


FIG. 5-11. The equivalent circuit of Fig. 5-10 with feedback removed.

To deduce the equivalent circuit of Fig. 5-10 with feedback present, note that the effect of feedback appears in the form of the potential $\mathbf{E}_g$. Without feedback, $\mathbf{E}_g = \mathbf{E}_1$. With feedback, $\mathbf{E}_g = \mathbf{E}_1 + \beta\mathbf{E}_2$. Clearly, from this discussion, the equivalent circuit of Fig. 5-10 with feedback present is that shown in Fig. 5-12. It should be noted that this figure, even though it is the equivalent circuit of Fig. 5-10 when feedback is present, is not a Thévenin potential-source equivalent representation, because both $\mathbf{Z}_t$ and $\mathbf{K}_t(\mathbf{E}_1 + \beta\mathbf{E}_2)$ are functions of the load. Note from the diagram that

$$\mathbf{E}_2 = \mathbf{K}_t(\mathbf{E}_1 + \beta\mathbf{E}_2) - \mathbf{I}\mathbf{Z}_t$$

Therefore

$$\mathbf{E}_2(1 - \beta\mathbf{K}_t) = \mathbf{K}_t\mathbf{E}_1 - \mathbf{I}\mathbf{Z}_t$$

or

$$\mathbf{E}_2 = \frac{\mathbf{K}_t}{1 - \beta\mathbf{K}_t}\mathbf{E}_1 - \frac{\mathbf{Z}_t}{1 - \beta\mathbf{K}_t}\mathbf{I} \tag{5-30}$$

The Thévenin equivalent network for the circuit with feedback is, according to this expression, that shown in Fig. 5-13.

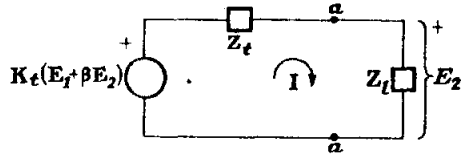


FIG. 5-12. The equivalent circuit of Fig. 5-10 with feedback present.

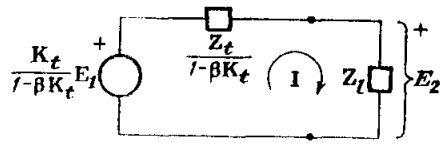


FIG. 5-13. The Thévenin equivalent network of Fig. 5-10.

As a check of these results, it is noted that $\mathbf{K}_t\mathbf{E}_1/(1 - \beta\mathbf{K}_t)$ represents the open-circuit potential with feedback, $\mathbf{K}_f\mathbf{E}_1$. This agrees with the result obtained in Eq. (5-4) that $\mathbf{K}_f = \mathbf{K}/(1 - \beta\mathbf{K})$. The internal impedance with potential feedback is

$$\mathbf{Z}_{tf} = \frac{\mathbf{Z}_t}{1 - \beta\mathbf{K}_t} \tag{5-31}$$

Since, for negative feedback, $1 - \beta\mathbf{K}_t$ is greater than unity, the impedance with feedback is less than that without feedback. Note, moreover, that the effective internal impedance is reduced by the same factor as the gain, when feedback is applied.

It is interesting to apply these results to the potential-feedback circuit of Fig. 5-5. If the load is open-circuited, then the magnitude of the gain of the circuit is simply the μ of the tube and $\mathbf{K}_t = -\mu$. Also, the internal impedance without feedback is r_p, whence $\mathbf{Z}_t = r_p$. The results so obtained agree with those in Fig. 5-6.

It is now desired to examine the results of current or series feedback

on the effective internal impedance. Refer to Fig. 5-14, which shows a general feedback circuit with current feedback. Although the impedance $\mathbf{Z}_f$ is shown isolated from the remainder of the circuit, it is a part of the feedback circuit and is not part of the external load $\mathbf{Z}_l$. The

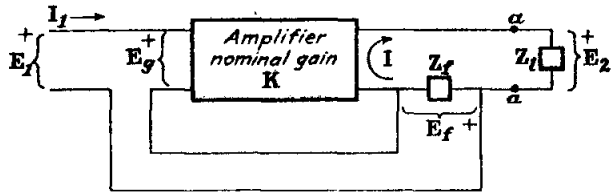


FIG. 5-14. The general current or series feedback circuit.

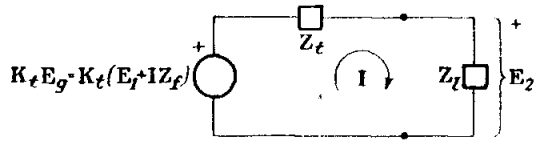


FIG. 5-15. The equivalent circuit of Fig. 5-14 with feedback present.

equivalent circuit will be of the form shown in Fig. 5-15. In this diagram $\mathbf{Z}_t$ is the total internal impedance looking back from the load and includes the effect of $\mathbf{Z}_f$. From Fig. 5-15 it follows that

$$\mathbf{K}_t(\mathbf{E}_1 + \mathbf{I}\mathbf{Z}_f) = \mathbf{I}(\mathbf{Z}_t + \mathbf{Z}_l)$$

from which

$$\mathbf{I} = \frac{\mathbf{K}_t\mathbf{E}_1}{\mathbf{Z}_t - \mathbf{Z}_f\mathbf{K}_t + \mathbf{Z}_l}$$

But this is the current that exists in the circuit of Fig. 5-16, which is the Thévenin equivalent with current feedback.

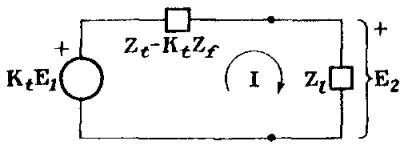


FIG. 5-16. The Thevenin equivalent of Fig. 5-14.

The effective internal impedance of the equivalent Thévenin generator with current feedback is thus seen to be

$$\mathbf{Z}_{tf} = \mathbf{Z}_t - \mathbf{K}_t\mathbf{Z}_f \qquad (5\text{-}32)$$

Note that the open-circuit potential with feedback is $\mathbf{K}_t\mathbf{E}_1$, which equals the open-circuit potential without feedback, in view of the significance of $\mathbf{K}_t$. This result is consistent with the observation that if $\mathbf{Z}_l$ is removed from Fig. 5-16, an open circuit results, and $\mathbf{I}$, and so the feedback, is zero.

These results are applied to Fig. 5-3. If the output is removed, the open-circuit gain is $\mathbf{K}_t = -\mu$. The internal impedance without feedback is $\mathbf{Z}_t = r_p + R_k$. Hence

$$\mathbf{Z}_{tf} = r_p + R_k + \mu R_k$$

which agrees with the previous result.

Example 1. Analyze the amplifier of Fig. 5-17 by the use of feedback methods, when the output is taken across R_l.

Solution. The equivalent circuit is given in Fig. 5-17*b*. It is first noted that the grid-cathode potential $\mathbf{E}_g$ is given by the expression

$$\mathbf{E}_g = \mathbf{E}_1 - \mathbf{E}_k$$

But since $\mathbf{E}_k = \mathbf{I}R_k$, then

$$\frac{\mathbf{E}_k}{\mathbf{I}} = R_k$$

which is independent of the load impedance. This indicates, according to the criterion given in Sec. 5-3, that current feedback exists.

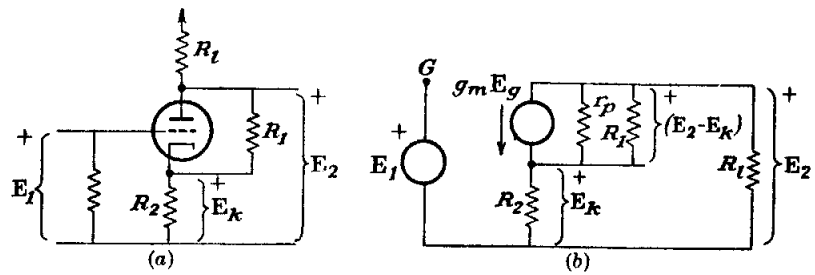


FIG. 5-17. The schematic and equivalent circuit of Example 1.

To analyze the circuit completely, it is desired to calculate both $\mathbf{K}_f$ and $\mathbf{Z}_{tf}$. To evaluate these requires an evaluation of β, $\mathbf{K}$, $\mathbf{K}_t$, and $\mathbf{Z}_t$. Note from the equivalent circuit that

$$\beta = -\frac{\mathbf{E}_k}{\mathbf{E}_2} = \frac{R_2}{R_l}$$

The nominal gain is found by noting that

$$g_m\mathbf{E}_g + (\mathbf{E}_2 - \mathbf{E}_k)\left(\frac{1}{r_p} + \frac{1}{R_l}\right) = -\frac{\mathbf{E}_2}{R_l}$$

But

$$\mathbf{E}_k = -\frac{\mathbf{E}_2}{R_l}R_2$$

Then

$$g_m\mathbf{E}_g + \mathbf{E}_2\left(1 + \frac{R_2}{R_l}\right)\left(\frac{1}{r_p} + \frac{1}{R_1}\right) + \frac{\mathbf{E}_2}{R_l} = 0$$

or

$$g_m\mathbf{E}_g + \mathbf{E}_2\left[\frac{1}{R_l} + \frac{1}{R_l}\frac{(R_l + R_2)(R_1 + r_p)}{r_pR_1}\right] = 0$$

Therefore

$$\mathbf{K} = \frac{\mathbf{E}_2}{\mathbf{E}_g} = \frac{-g_m}{\frac{1}{R_l}\left[\frac{r_pR_1 + (R_l + R_2)(R_1 + r_p)}{r_pR_1}\right]} \qquad (5\text{-}33)$$

The equivalent gain on open circuit is simply

$$\mathbf{K}_t = \frac{-\mu R_1}{r_p + R_1} \tag{5-34}$$

Also the equivalent impedance $\mathbf{Z}_t$ is

$$\mathbf{Z}_t = R_2 + \frac{r_p R_1}{r_p + R_1} = \frac{(r_p + R_1)R_2 + r_p R_1}{r_p + R_1} \tag{5-35}$$

It follows from these expressions that the gain with feedback is

$$\mathbf{K}_f = \frac{\dfrac{-g_m R_l}{\dfrac{r_p R_1 + (R_l + R_2)(R_1 - r_p)}{r_p R_1}}}{1 + \dfrac{R_2}{R_1}\dfrac{g_m R_l}{\dfrac{r_p R_1 + (R_l + R_2)(R_1 + r_p)}{r_p R_1}}}$$

This reduces to

$$\mathbf{K}_f = \frac{-\mu R_l R_1}{r_p(R_1 + R_2 + R_l) + (\mu + 1)R_1 R_2 + R_1 R_l} \tag{5-36}$$

Also, the effective internal impedance is

$$\mathbf{Z}_{tf} = \mathbf{Z}_t - \mathbf{K}_t \mathbf{Z}_f$$

$$= \frac{(r_p + R_1)R_2 + r_p R_1}{r_p + R_1} + \frac{\mu R_1}{r_p + R_1} R_2$$

$$\mathbf{Z}_{tf} = \frac{r_p(R_1 + R_2) + (\mu + 1)R_1 R_2}{r_p + R_1} \tag{5-37}$$

It is of some interest to examine the effects of the feedback on the gain and on the effective internal impedance. The gain ratio, given by the ratio of Eq. (5-36) to Eq. (5-33), is found to be

$$\frac{\mathbf{K}_f}{\mathbf{K}} = \frac{r_p(R_1 + R_2 + R_l) + R_1 R_l + R_1 R_2}{r_p(R_1 + R_2 + R_l) + R_1 R_l + (\mu + 1)R_1 R_2}$$

which may be written in the form

$$\frac{\mathbf{K}_f}{\mathbf{K}} = \frac{1}{1 + \dfrac{\mu R_1 R_2}{r_p(R_1 + R_2 + R_l) + R_1 R_l + R_1 R_2}}$$

This expression shows that the resultant gain with feedback is less than that without feedback, as expected.

In a somewhat similar way, the effective internal impedances may be compared, to examine the effects of the feedback. By Eqs. (5-37) and (5-35), the ratio is readily found to be

$$\frac{\mathbf{Z}_{tf}}{\mathbf{Z}_t} = 1 + \frac{\mu R_1 R_2}{r_p(R_1 + R_2) + R_1 R_2}$$

The effect of the feedback is to increase the effective internal impedance, which is characteristic of current feedback.

Example 2. Analyze the circuit of Example 1 when the output is taken across the cathode resistor R_2.

Solution. In the present case, since $\mathbf{E}_k$ is the output potential, then since

$$\mathbf{E}_g = \mathbf{E}_1 - \mathbf{E}_k$$

it follows that

$$\beta = -1$$

But since β is independent of the load, then potential feedback now exists.

The nominal gain is obtained from a study of the equivalent circuit of Fig. 5-17*b*. It is observed that

$$g_m \mathbf{E}_g + (\mathbf{E}_2 - \mathbf{E}_k)\left(\frac{1}{r_p} + \frac{1}{R_1}\right) = \frac{\mathbf{E}_k}{R_2}$$

Also

$$\mathbf{E}_2 = -\frac{R_l}{R_2}\mathbf{E}_k$$

Then

$$g_m \mathbf{E}_g - \mathbf{E}_k\left(1 + \frac{R_l}{R_2}\right)\left(\frac{1}{r_p} + \frac{1}{R_1}\right) - \frac{\mathbf{E}_k}{R_2} = 0$$

or

$$g_m \mathbf{E}_g - \mathbf{E}_k\left[\frac{1}{R_2} + \frac{1}{R_2}\frac{(R_2 + R_l)(R_1 + r_p)}{r_p R_1}\right] = 0$$

Therefore

$$\mathbf{K} = \frac{\mathbf{E}_k}{\mathbf{E}_g} = \frac{g_m}{\dfrac{1}{R_2}\dfrac{r_p R_1 + (R_l + R_2)(R_1 + r_p)}{r_p R_1}}$$

which is

$$\mathbf{K} = \frac{\mu R_1 R_2}{r_p R_l + (R_l + R_2)(R_1 + r_p)} \tag{5-38}$$

The equivalent gain on open circuit is

$$\mathbf{K}_t = \frac{\mu R_1}{r_p + R_1} \tag{5-39}$$

Also the equivalent impedance $\mathbf{Z}_t$ is

$$\mathbf{Z}_t = R_l + \frac{r_p R_1}{r_p + R_1} = \frac{(r_p + R_1)R_l + r_p R_1}{r_p + R_1} \tag{5-40}$$

It follows from these expressions that under feedback conditions

$$\mathbf{K}_f = \frac{\mu R_1 R_2/[r_p R_1 + (R_l + R_2)(R_1 + r_p)]}{1 + \mu R_1 R_2/[r_p R_1 + (R_l + R_2)/(R_1 + r_p)]}$$

which reduces to

$$\mathbf{K}_f = \frac{\mu R_1 R_2}{r_p(R_1 + R_2 + R_l) + (\mu + 1)R_1 R_2 + R_1 R_l} \tag{5-41}$$

The corresponding effective internal impedance is

$$\mathbf{Z}_{tf} = \frac{\mathbf{Z}_t}{1 - \beta\mathbf{K}_t} = \frac{\dfrac{(r_p + R_1)R_l + r_pR_1}{r_p + R_1}}{1 + \dfrac{\mu R_1}{r_p + R_1}}$$

or

$$\mathbf{Z}_{tf} = \frac{(r_p + R_1)R_l + r_pR_1}{r_p + (\mu + 1)R_1} \tag{5-42}$$

While it is possible to draw certain conclusions from a comparison of the results obtained in Examples 1 and 2, the same conclusions are possible from the simplified circuit illustrated in Fig. 5-18, in which $R_l = R_2$, and R_1 is set to infinity, or an open circuit. This circuit is known as a single-tube "paraphase" amplifier and provides two equal output potentials of opposite polarity from a single excitation source. For the case when the output potential is $\mathbf{E}_2$, the significant expressions deduced from Eqs. (5-36) and (5-37) are the following:

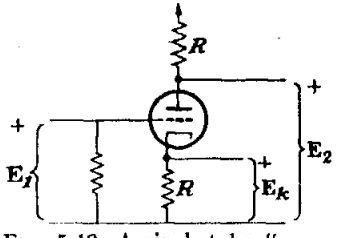


FIG. 5-18. A single-tube "paraphase" amplifier.

$$\mathbf{K}_f = \frac{-\mu R_l}{r_p + (\mu + 2)R_l} \qquad \mathbf{Z}_{tf} = r_p + (\mu + 1)R_l \tag{5-43}$$

When the output potential is $\mathbf{E}_k$, the appropriate expressions become, from Eqs. (5-41) and (5-42),

$$\mathbf{K}_f = \frac{\mu R_l}{r_p\ (\mu + 2)R_l} \qquad \mathbf{Z}_{tf} = \frac{r_p + R_l}{\mu + 1} \tag{5-44}$$

It will be observed that the gain of the amplifier with respect to each output pair of terminals is the same. However, it is also noted that the effective internal impedances looking back from these terminals are quite different, one being much higher than the other.

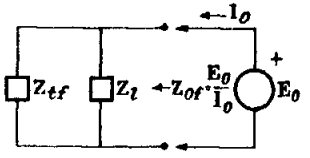


FIG. 5-19. The output-terminal impedance of the general feedback amplifier.

5-5. Effect of Feedback on the Output-terminal Impedance. The output-terminal impedance of a circuit is the impedance looking back into the output terminals of the network when the load impedance is in place, but with the input potential reduced to zero. Clearly, the output impedance of an amplifier is the parallel combination of the effective internal impedance and the load impedance. Since the equivalent internal impedance $\mathbf{Z}_{tf}$ depends on the type of feedback that is incorporated in the amplifier, then the output impedance will also depend on the type of feedback. The situation is illustrated schematically in Fig. 5-19.

The output impedance $\mathbf{Z}_{0f}$, which is given as the ratio of the current $\mathbf{I}_0$ into the output terminals when a potential $\mathbf{E}_0$ is impressed, is clearly

$$\mathbf{Z}_{0f} = \frac{\mathbf{Z}_{tf}\mathbf{Z}_l}{\mathbf{Z}_{tf} + \mathbf{Z}_l} \tag{5-45}$$

where, for the case of potential feedback, by Eq. (5-31)

$$\mathbf{Z}_{tf} = \frac{\mathbf{Z}_t}{1 - \beta\mathbf{K}_t}$$

and for the case of current feedback, by Eq. (5-32), is

$$\mathbf{Z}_{tf} = \mathbf{Z}_t - \mathbf{K}_f\mathbf{Z}_f$$

It is desired to obtain an expression for $\mathbf{Z}_{0f}$ in terms of the output impedance without feedback, Z_0, where

$$\mathbf{Z}_0 = \frac{\mathbf{Z}_t\mathbf{Z}_l}{\mathbf{Z}_t + \mathbf{Z}_l} \tag{5-46}$$

a. Potential Feedback. Suppose that the input source to the general feedback amplifier is reduced to zero and that a potential source is applied

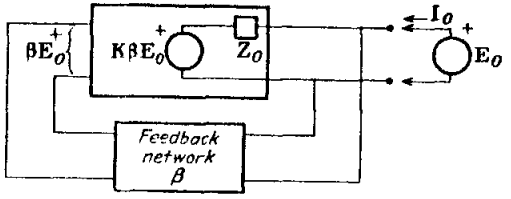


FIG. 5-20. The modifications of the general potential feedback amplifier for calculating the output-terminal impedance.

to the output terminals. The situation is illustrated in Fig. 5-20. This diagram is Fig. 5-1 appropriately modified for output-impedance determinations. In view of Fig. 5-11, which gives the equivalent circuit of the general amplifier with potential feedback, then $\mathbf{Z}_0$ has the form of Eq. (5-46).

The current $\mathbf{I}_0$ from the applied source is seen to be

$$\mathbf{I}_0 = \frac{\mathbf{E}_0 - \mathbf{K}\beta\mathbf{E}_0}{\mathbf{Z}_0}$$

and the effective output impedance with feedback is

$$\mathbf{Z}_{0f} = \frac{\mathbf{E}_0}{\mathbf{I}_0} = \frac{\mathbf{Z}_0}{1 - \mathbf{K}\beta} \tag{5-47}$$

which is similar in form to Eq. (5-4). This shows that the output impedance is reduced by the same factor as the potential gain with the application of potential feedback.

b. Current Feedback. The calculation for the output impedance of an amplifier which employs current feedback follows a similar pattern. In this case, as before, the input signal is reduced to zero, and a potential source is applied to the output terminals. The current-feedback circuit for the output-terminal impedance calculation then becomes that shown in Fig. 5-21.

In this circuit $\mathbf{Z}_0$ denotes the output impedance of the circuit without feedback and includes the effect of $\mathbf{Z}_f$. $\mathbf{K}$ is the gain without feedback,

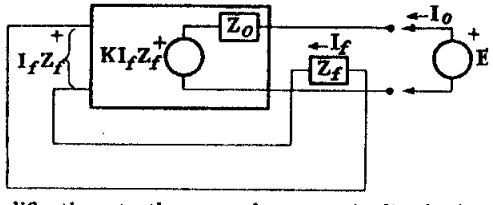


FIG. 5-21. The modifications to the general current feedback circuit for output-terminal impedance determination.

but with $\mathbf{Z}_l$ in position. The potential $\mathbf{E}_g$ is the drop across $\mathbf{Z}_f$ and is $\mathbf{I}_f\mathbf{Z}_f$.

It follows from the diagram, by taking account of the current through the load impedance, that

$$\mathbf{E}_0 = \mathbf{I}_0\mathbf{Z}_0 - \mathbf{K}\mathbf{I}_f\mathbf{Z}_f$$

But this becomes

$$\mathbf{E}_0 = \mathbf{I}_0\mathbf{Z}_0 - \mathbf{K}\mathbf{Z}_f\left(\mathbf{I}_0 - \frac{\mathbf{E}_0}{\mathbf{Z}_l}\right)$$

This gives

$$\mathbf{I}_0(\mathbf{Z}_0 - \mathbf{K}\mathbf{Z}_f) = \mathbf{E}_0\left(1 - \mathbf{K}\frac{\mathbf{Z}_f}{\mathbf{Z}_l}\right)$$

from which it follows that the effective output impedance with feedback is

$$\mathbf{Z}_{0f} = \frac{\mathbf{Z}_0 - \mathbf{K}\mathbf{Z}_f}{1 - \mathbf{K}(\mathbf{Z}_f/\mathbf{Z}_l)} = \mathbf{Z}_0\frac{1 - \mathbf{K}(\mathbf{Z}_f/\mathbf{Z}_0)}{1 - \mathbf{K}(\mathbf{Z}_f/\mathbf{Z}_l)} \tag{5-48}$$

5-6. Effect of Feedback on the Input-terminal Impedance. It is of some importance to examine how the input impedance of an amplifier is affected by the presence of feedback. It will be found that the effective input impedance increases for both potential and current feedback.

a. Potential Feedback. It follows directly from Fig. 5-10 that the input-terminal impedance with feedback is simply

$$\mathbf{Z}_{1f} = \frac{\mathbf{E}_1}{\mathbf{I}_1}$$

This may be written as

$$\mathbf{Z}_{1f} = \frac{\mathbf{E}_g - \beta\mathbf{E}_2}{\mathbf{I}_1} = \frac{\mathbf{E}_g}{\mathbf{I}_1}(1 - \mathbf{K}\beta)$$

But the input impedance without feedback is

$$\mathbf{Z}_1 = \frac{\mathbf{E}_g}{\mathbf{I}_1}$$

Then

$$\mathbf{Z}_{1f} = \mathbf{Z}_1(1 - \mathbf{K}\beta) \tag{5-49}$$

Therefore, owing to the feedback, the input impedance with feedback is greater than the input impedance without feedback, and in the same degree as the gain and distortion decrease.

As a specific example, suppose that $\mathbf{Z}_1$ is the impedance due to a capacitance between the grid-cathode terminals, and this may be the actual tube capacitance modified by the Miller effect. Since the impedance increases with feedback, this means that the effective input capacitance is decreased. Clearly, therefore,

$$C_{1f} = \frac{C_1}{1 - K} \tag{5-50}$$

b. Current Feedback. By proceeding as in (*a*) for the potential feedback, but now with reference to Fig. 5-14 for the general current-feedback circuit,

$$\mathbf{Z}_{1f} = \frac{\mathbf{E}_1}{\mathbf{I}_1} = \frac{\mathbf{E}_g - \mathbf{I}\mathbf{Z}_f}{\mathbf{I}_1} = \left(\mathbf{E}_g - \frac{\mathbf{E}_2}{\mathbf{Z}_l}\mathbf{Z}_f\right)\frac{1}{\mathbf{I}_1}$$

But

$$\beta = \frac{\mathbf{Z}_f}{\mathbf{Z}_l} \qquad \mathbf{E}_2 = \mathbf{K}\mathbf{E}_g$$

Then it follows that

$$\mathbf{Z}_{1f} = \mathbf{Z}_1(1 - \mathbf{K}\beta) \tag{5-51}$$

Note that the input impedance with current feedback is greater than the input impedance without feedback in the same degree as for the case of potential feedback.

5-7. Feedback and Stability. A great deal of information about the stability of an amplifier can be obtained from an analysis of the factor $1 - \mathbf{K}\beta$ that appears in the general gain expression [Eq. (5-3)]. This is best analyzed through the use of the polar plot of the expression $\mathbf{K}\beta$. Attention is first called to the significance of the quantity $\mathbf{K}\beta$. This is best examined by reference to the diagram of Fig. 5-22. Observe that $\mathbf{K}\beta$ is the total open-loop gain, including the amplifier and the feedback

network, but with the feedback connection open. In network parlance, this is the open-loop transfer function of the amplifier and the feedback network. In essence, therefore, consideration of the open-loop performance of the amplifier and feedback network is to be used to provide significant information regarding the performance of the amplifier under closed-loop operation.

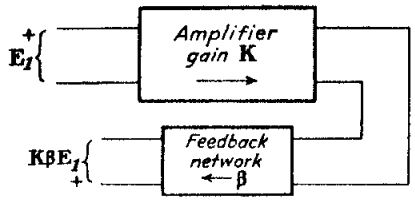


FIG. 5-22. To illustrate the significance of the factor Kβ.

Kβ is a function of the frequency, and, in general, points in the complex plane are obtained for the values of Kβ corresponding to all values of f from 0 to ∞. The locus of all these points forms a closed curve.

As a particular example, suppose that the locus of Kβ in the complex plane is drawn for the amplifier illustrated in Fig. 5-5. To do this, the complete expression for the nominal gain, including the effect of the feedback circuit, must be written, rather than the simple form given in Eq. (5-23). Also, the value of β must include the effects of the blocking capacitor C. Certain of the features of the response of this amplifier are known. At the mid-frequencies, the gain is substantially constant and has a phase of 180 deg. For the low and high frequencies, the gain falls

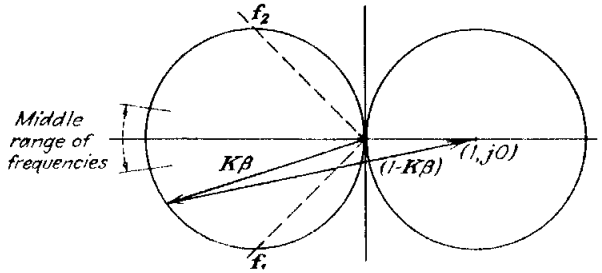


FIG. 5-23. The locus in the complex plane of Kβ for the circuit of Fig. 5-5.

to zero, and the phase approaches ±90 deg, respectively. At the l-f and h-f cutoff values the phase is ±135 deg, respectively. It may be shown that the general locus of Kβ of this amplifier for all frequencies is a circle. The result is shown in Fig. 5-23.

Suppose that a phasor is drawn from the polar locus to the point (1,j0). This is the quantity $1 - K\beta$, as shown. For this particular case, its magnitude is greater than unity for all frequencies, and it has its maximum magnitude at the middle range of frequencies. Moreover, since the resultant gain varies inversely with $1 - K\beta$, then the effect of the feedback is to cause a general flattening of the frequency-response characteristic.

The criterion for positive and negative feedback is evident on the complex plane. First note that the quantity $|1 - K\beta| = 1$ represents a circle of unit radius with its center at the point (1,j0), as illustrated. Clearly, if for a given amplifier $|1 - K\beta| > 1$, then the feedback is negative, with an over-all reduction of gain. Likewise, if $|1 - K\beta| < 1$, there is an over-all increase in gain and the feedback is positive. These considerations show that if Kβ extends outside of the unit circle for any frequency, then the feedback is negative at that frequency. If Kβ lies within the unit circle, then the feedback is positive. If Kβ passes through the point (1,j0) then $1 - K\beta = 0$, and, as will later be shown, the amplifier is unstable and oscillates. A more general analysis by Nyquist[1,3] shows that the amplifier will oscillate if the curve Kβ encloses the point (1,j0) and is stable if the curve does not enclose this point. That is, if the magnitude of Kβ is less than unity when its phase angle is zero, no oscillations are possible.

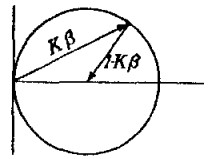


FIG. 5-24. A plot of $|1 - K\beta| = 1$ in the complex plane.

As a specific example for discussion, suppose that the plot of a given amplifier is that illustrated in Fig. 5-25. The feedback is negative for this amplifier in the frequency range from 0 to f_1. Positive feedback exists in the frequency range from f_1 to ∞. Note, however, that since the locus of Kβ does not enclose the point (1,j0), then, according to the Nyquist criterion, oscillations will not occur.

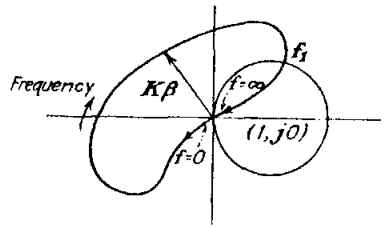


FIG. 5-25. The polar plot of an amplifier.

5-8. Transistor Stability Considerations. Stability of a transistor circuit is assured if the input and internal resistances are always positive. Of course, a negative value of either does not necessarily imply oscillation. On the other hand, for completely stable operation, positive values of both R_i and R_t provide a sufficient condition. It is therefore of interest to examine the expressions for input and internal resistances for both junction and point-contact transistors.

For junction transistors under normal operation, the value of a is always less than unity. Thus $r_c - r_m = r_c(1 - a)$ is always positive. An examination of the expressions for R_i and R_t in Tables 3-1 to 3-3 indicates that these have positive values for all positive values of R_l and R_g. If the driving or load impedances are reactive, a calculation for $\mathbf{Z}_i$ and $\mathbf{Z}_t$ will have positive real parts. Consequently, for the low frequencies, the junction transistor is unconditionally stable.

The situation for the point-contact transistors is quite different, since a exceeds unity, in general; hence $r_c - r_m$ is ordinarily negative. As a

result, there is a region of instability. The situation is examined in detail, for the grounded-base connection.

For a positive input resistance of the grounded-base connection, it is necessary that R_i in Eq. (3-35) be positive. Thus

$$r_e + r_b \frac{r_c - r_m + R_l}{r_b + r_c + R_l} > 0$$

But $r_b + r_c + R_l > 0$, whence

$$r_e(r_b + r_c + R_l) + r_b(r_c + R_l) - r_b r_m > 0$$

which becomes, by dividing by $r_b(r_c + R_l)$,

$$\frac{r_e}{r_c + R_l} + \frac{r_e}{r_b} + 1 - \frac{r_m}{r_c + R_l} > 0$$

or

$$1 + \frac{r_e}{r_b} > \frac{r_m - r_e}{r_c + R_l} \tag{5-52}$$

This expression shows that, for input stability, r_b should be as small as possible; R_l should be as high as possible; or add external resistance to r_e to increase this effective value.

For positive internal resistance, from Eq. (3-37), it is required that

$$r_c - r_b \frac{r_m - R_g - r_e}{R_g + r_e + r_b} > 0$$

Write this as

$$r_c(R_g + r_e + r_b) + r_b(R_g + r_e) - r_b r_m > 0$$

Divide by $r_c r_b$ to get

$$\frac{R_g + r_e}{r_b} + 1 - \frac{r_m}{r_c} + \frac{R_g + r_e}{r_c} > 0$$

or

$$1 + (R_g + r_c)\left(\frac{1}{r_b} + \frac{1}{r_e}\right) > \frac{r_m}{r_c} \tag{5-53}$$

To ensure a positive output resistance, r_b should be small; also, $R_g + r_e$ should be large.

The conditions for stability of the other connections are given without proof in Table 5-1. Observe from this table that the addition of resistance in the base lead will ensure a positive input resistance from the grounded-emitter and the grounded-collector circuits. Also, for a positive internal impedance, $R_g + r_b$ should be kept small, or r_e should be made large. The first of these conditions indicates that the cascading of grounded-emitter or grounded-collector stages might prove troublesome. It should perhaps be noted that this is one of several reasons why point-contact transistors are never used in amplifier applications. Another reason is to be found in the inherently higher noise figure of the point-contact transistor, being roughly 60 db above theoretical, whereas the junction transistor is roughly 20 db above theoretical.

TABLE 5-1

CONDITIONS OF STABILITY

Connection	For positive input resistance	For positive internal resistance
Grounded-base	$1 + \frac{r_e}{r_b} > \frac{r_m - r_e}{r_c + R_l}$	$1 + (R_g + r_e)\left(\frac{1}{r_b} + \frac{1}{r_c}\right) > \frac{r_m}{r_c}$
Grounded-emitter	$r_b > -\frac{r_e}{1 - \frac{r_m - r_e}{r_c + r_l}}$	$1 + \frac{r_e}{R_g + r_b} + \frac{r_e}{r_c} > \frac{r_m}{r_c}$
Grounded-collector	$r_b > -r_c \frac{r_c + R_l}{r_c - r_m + R_l + r_e}$	$1 + \frac{r_e}{R_g + r_b} + \frac{r_e}{r_c} > \frac{r_m}{r_c}$

5-9. The Cathode Follower.[4] The cathode follower is illustrated in Fig. 5-26*a*, the equivalent circuit being given in Fig. 5-26*b*. This feedback circuit is singled out for detailed consideration because of its extensive use in a variety of applications. These applications stem from the

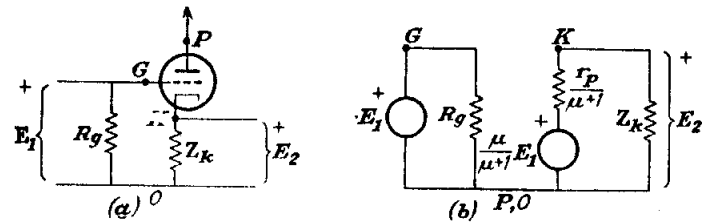


FIG. 5-26. Schematic and equivalent circuit of the cathode follower.

fact that the cathode follower possesses a high input impedance and a low output impedance and may therefore be used as a coupling device between a high impedance source and a low impedance load.

The cathode follower is similar to the single-tube paraphase amplifier of Fig. 5-18, but with a zero plate load. The equivalent circuit is that shown in Fig. 5-26*b*. The grid circuit is isolated, since interelectrode capacitances are neglected, and the input impedance is R_g. The effective internal impedance is $\mathbf{Z}_{tf} = r_p/(\mu + 1)$, which is very low. In fact, if $\mu \gg 1$, then $\mathbf{Z}_{tf} = r_p/\mu = 1/g_m$. But since g_m for most tubes varies from 1,000 to 10,000 μmhos, then $\mathbf{Z}_{tf}$ is of the order of 1,000 to 100 ohms. A double cathode-follower circuit has been devised which has a greatly reduced effective internal impedance.[5]

The gain is obtained from an analysis of Fig. 5-26*b* and is given by

$$\mathbf{K} = \frac{\mu \mathbf{Z}_k}{r_p + (\mu + 1)\mathbf{Z}_k} = \frac{\mu}{(\mu + 1) + (r_p/\mathbf{Z}_k)} \tag{5-54}$$

Clearly **K** approaches the limiting value $\mu/(\mu + 1)$ as the ratio $r_p/\mathbf{Z}_k$ approaches zero, or as $\mathbf{Z}_k \gg r_p$. For tubes with large value of μ, and with $\mathbf{Z}_k \gg r_p$, the gain approaches unity. For values of $\mathbf{Z}_k$ and r_p found in most normal cases, **K** is of the order of 0.9 or higher. In fact, it is because of this unity-gain feature that the circuit derives its name, since the output potential is almost equal to the input potential, whence the

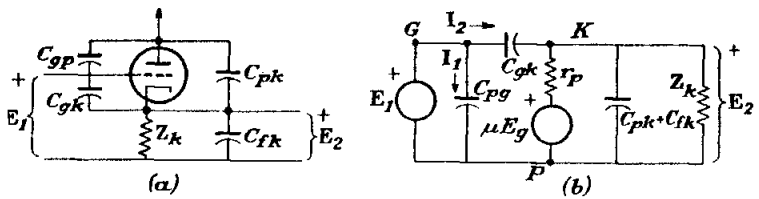


FIG. 5-27. Schematic and equivalent circuits of the cathode follower including the interelectrode capacitances.

cathode and grid rise and fall together in potential by almost equal amounts (or the cathode follows the grid).

The interelectrode and wiring capacitances have been neglected in the above discussions, as the effects of these are usually negligible for frequencies below about 1 Mc/sec. For purposes of our study, these will be taken into account. The schematic and equivalent circuits are now given in Fig. 5-27.

The expression for the gain of the amplifier is deduced by analyzing the circuit. It is noted that

$$\mathbf{E}_2 = \frac{\mathbf{E}_1 \mathbf{Y}_{C_{gk}} + \mu \mathbf{E}_g \mathbf{Y}_p}{\mathbf{Y}_{C_{gk}} + \mathbf{Y}_p + \mathbf{Y}_{C_{pk}+C_{fk}} + \mathbf{Y}_{Z_k}} \tag{5-55}$$

But it follows that

$$\mathbf{E}_g = \mathbf{E}_1 - \mathbf{E}_2$$

and Eq. (5-55) becomes

$$\mathbf{E}_2 = \frac{j\omega C_{gk}\mathbf{E}_1 + \mu \mathbf{Y}_p(\mathbf{E}_1 - \mathbf{E}_2)}{j\omega(C_{gk} + C_{pk} + C_{fk}) + 1/r_p + 1/\mathbf{Z}_k} \tag{5-56}$$

Solving for the gain $\mathbf{K}_f$ which is given by $\mathbf{K}_f = \mathbf{E}_2/\mathbf{E}_1$, there results

$$\mathbf{K}_f = \frac{(j\omega r_p C_{gk} + \mu)\mathbf{Z}_k}{j\omega r_p \mathbf{Z}_k(C_{gk} + C_{pk} + C_{fk}) + r_p + (\mu + 1)\mathbf{Z}_k} \tag{5-57}$$

For those values of $\mathbf{Z}_k$ which are normally used, the effect of the interelectrode and wiring capacitances on the potential amplification is negligible for frequencies below about 1 Mc, as already noted. That this is so is seen by writing Eq. (5-57) in the form

$$\mathbf{K}_f = \frac{(g_m + j\omega C_{gk})\mathbf{Z}_k}{1 + \left(\frac{\mu + 1}{r_p} + j\omega C_T\right)\mathbf{Z}_k} \tag{5-58}$$

where $C_T = C_{gk} + C_{pk} + C_{fk}$. But the effect of the capacitances will become important only for those frequencies for which ωC_T becomes comparable with $(\mu + 1)/r_p \doteq g_m$. If C_T is taken as 30 $\mu\mu$f and $g_m = 1{,}000$ μmhos, then $f \doteq g_m/2\pi C_T \doteq 5$ Mc.

To find an expression for the input capacitance of the cathode follower, refer to Fig. 5-27*b*. It is seen that the current flowing through the source comprises two components. One of these is the current through the capacitance C_{gp} and is

$$\mathbf{I}_1 = j\omega C_{gp}\mathbf{E}_1 \tag{5-59}$$

The second is the current through the capacitance C_{gk}. This is

$$\mathbf{I}_2 = j\omega C_{gk}\mathbf{E}_g \tag{5-60}$$

But as $\mathbf{E}_g = \mathbf{E}_1 - \mathbf{E}_2$ and $\mathbf{K}_f = \mathbf{E}_2/\mathbf{E}_1$, then

$$\mathbf{I}_2 = j\omega C_{gk}(1 - \mathbf{K}_f)\mathbf{E}_1 \tag{5-61}$$

The total current is

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = j\omega[C_{gp}\mathbf{E}_1 + (1 - \mathbf{K}_f)C_{gk}\mathbf{E}_1]$$

and the effective input capacitance is

$$C_i = C_{gp} + (1 - \mathbf{K}_f)C_{gk} \tag{5-62}$$

Since in many circuits K_f is approximately 0.9, then C_i has the approximate value

$$C_i = C_{gp} + 0.1C_{gk} \tag{5-63}$$

A comparison of this expression with the corresponding form given by Eq. (3-16) for the conventional amplifier stage shows a roughly similar dependence on the tube capacitances, although the numerical value for the cathode follower is considerably smaller than that for the conventional amplifier stage.

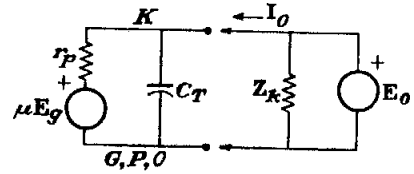


FIG. 5-28. The equivalent output circuit of the cathode follower.

The effective internal impedance $\mathbf{Z}_{tf}$ can be determined by finding the current $\mathbf{I}_0$ as a consequence of the application of an a-c potential $\mathbf{E}_0$ to the output terminals of Fig. 5-27. The grid exciting potential is made zero. The equivalent circuit is that drawn as Fig. 5-28, if the internal

impedance of the grid driving source is low. The effective internal admittance of the tube alone is found from

$$\mathbf{I}_0 = \mathbf{E}_0\mathbf{Y}_T + \frac{\mathbf{E}_0 - \mu\mathbf{E}_g}{r_p} \tag{5-64}$$

But under the conditions specified

$$\mathbf{E}_g = -\mathbf{E}_0$$

Then

$$\mathbf{Y}_{tf} = \frac{\mathbf{I}_0}{\mathbf{E}_0} = \mathbf{Y}_T + \mathbf{Y}_p + g_m \tag{5-65}$$

where $\mathbf{Y}_T = j\omega C_T$. It is of interest to compare this result with that which applies without capacitances being considered, viz.,

$$\mathbf{Y}_{tf} = \frac{1}{\mathbf{Z}_{tf}} = \frac{\mu + 1}{r_p} = g_m + \mathbf{Y}_p$$

The effect of the interelectrode capacitances is the addition of the term $\mathbf{Y}_T$ to the terms $\mathbf{Y}_p + g_m$. Here, as for the gain, $\mathbf{Y}_T$ does not become comparable with the other terms except at the higher video frequencies.

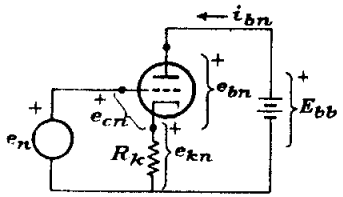


FIG. 5-29. The cathode follower with a cathode resistor.

5-10. Graphical Analysis of the Cathode Follower. Suppose that the cathode impedance is a resistance R_k, and this is the usual situation. A graphical solution of the operation of the circuit is possible on the plate characteristics of the tube. This necessitates drawing the dynamic characteristic of the circuit from the known plate characteristics. Refer to Fig. 5-29 for notation. The controlling equations of the grid and plate circuits are

$$e_n = e_{cn} + i_{bn}R_k \tag{5-66}$$

and

$$E_{bb} = e_{bn} + i_{bn}R_k \tag{5-67}$$

Equation (5-67) is the equation of the load line for the plate supply E_{bb} and the load resistance R_k. The procedure for constructing the dynamic characteristic follows:

1. On the plate characteristics draw the load line specified by Eq. (5-67). This is illustrated in Fig. 5-30.

2. Note the plate current at each point of intersection of the load line with the plate characteristics. For example, the current at the intersection of the load line with E_{c2} is labeled i_{b2}.

3. Now relabel the plate characteristics with the appropriate symbol e, according to Eq. (5-66). Thus

$$E_{c1} \rightarrow e_1(\equiv E_{c1} + i_{b1}R_k)$$
$$E_{c2} \rightarrow e_2(\equiv E_{c2} + i_{b2}R_k)$$

etc.

4. The dynamic characteristic is a plot of the (i_{bn},e_n) characteristic, where i_{bn} is the current corresponding to the input e_n. This requires calculating the value of e_n for each value of E_{cn}, and its corresponding i_{bn}.

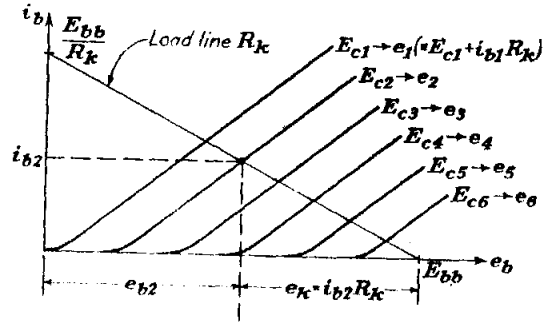


FIG. 5-30. Graphical construction for finding the dynamic characteristic of a cathode follower with cathode resistor R_k.

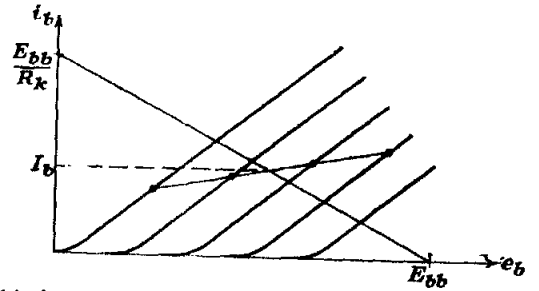


FIG. 5-31. Graphical construction for obtaining the value of I_b for a specified input potential E_1.

Often the complete dynamic curve is not required, but only the current I_b for a specified value of e, say E. By Eq. (5-66) this is

$$I_b = \frac{E - E_c}{R_k}$$

For several values of e_c (and the available E_c values are used), the value of i_b is calculated, and noted on the plate characteristics, as shown in Fig. 5-31. The intersection of the load line and the line connecting the calculated points is the appropriate current I_b for the specified E.

It should be specifically noted that the value of the input signal E will be quite large before E_c, the actual grid-cathode potential, becomes posi-

tive with the consequent grid current. That is, since the cathode potential follows the grid potential rather closely (for a gain almost equal to unity), the input signal may swing considerably positive before the onset of grid current. The larger the value of R_k, the larger will be the allowable positive swing. When cutoff occurs, no potential difference appears across R_k. Consequently, the applied signal required to reach cutoff is independent of R_k.

Example. Consider a 6J5 tube with $E_{bb} = 300$ volts and $R_k = 10{,}000$ ohms. Find the maximum positive and negative input swings for positive grid-cathode potential and cutoff, respectively.

Solution. From the plate characteristics of the 6J5 (see Appendix B-9) and the specified E_{bb} and R_k, the following data are found:

$$\begin{aligned} &\text{For } E_c = 0: \quad I_b = 15.7 \text{ ma} \\ &\text{For } E_c = -18: \quad I_b = 0 \end{aligned}$$

This shows that the cathode follower may swing from +157 volts to −18 volts without drawing grid current or driving the tube beyond cutoff.

Clearly, the operation of the cathode-follower circuit of Fig. 5-29 is unsymmetrical. For small potential excursions, this causes no difficulty.

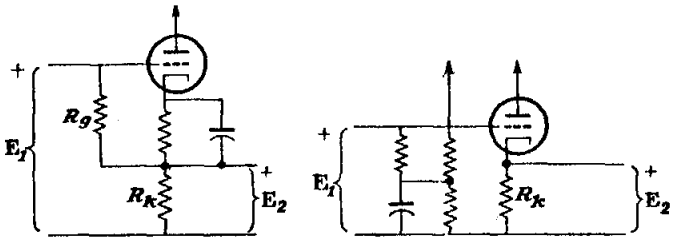


FIG. 5-32. Two ways of achieving more symmetrical operation of a cathode follower.

Also, if only positive signals are to be used, no difficulty exists. However, if large negative signals are to be applied, it is necessary to establish the grid at a large positive potential with respect to the bottom end of R_k (ordinarily ground), although the actual tube bias E_c will still be negative. This bias may be achieved in several ways, as illustrated in Fig. 5-32. For symmetrical operation, the bias will be established to set the d-c level across R_k at about half of the peak-peak potential swing.

5-11. The Anode Follower. A circuit which possesses roughly the same gain and impedance properties as the cathode follower, but which also provides phase reversal between the output and input potentials, is often called an *anode follower*. This is perhaps an unfortunate choice of name, since, unlike the cathode follower in which the cathode-potential variations follow the grid-potential variations rather closely, i.e., these potential variations are in the same phase, the anode of the anode follower falls when the grid potential rises. Clearly, no "following" occurs. Because of the almost equal potential changes in the grid and plate potentials, but with the reverse phase, the circuit has been called the *seesaw* circuit. Whatever the most suitable choice of name, the circuit is of value and importance.

The circuit of the anode follower is given in Fig. 5-33. Actually, this is one adaptation of a general operational feedback amplifier, other applications being examined in considerable detail in Chap. 8. It is a simple

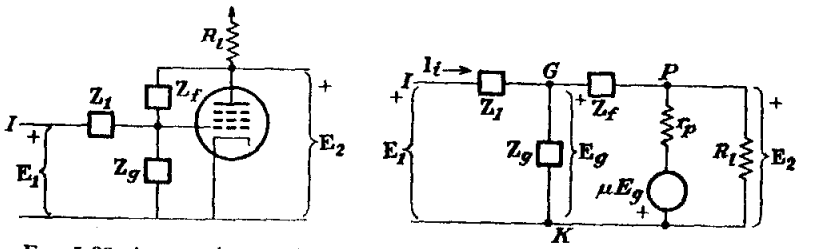


FIG. 5-33. A general potential feedback amplifier and its equivalent circuit.

amplifier which is provided with potential feedback through the impedance $\mathbf{Z}_f$. An analysis of the equivalent circuit is readily carried out. An application of the Millman network theorem between the points K and G yields

$$\mathbf{E}_g = \frac{\mathbf{E}_1\mathbf{Y}_1 + \mathbf{E}_2\mathbf{Y}_f}{\mathbf{Y}_1 + \mathbf{Y}_f + \mathbf{Y}_g} \tag{5-68}$$

When applied between K and P, the network theorem yields

$$\mathbf{E}_2 = \frac{\mathbf{E}_g\mathbf{Y}_f - \mu\mathbf{E}_g\mathbf{Y}_p}{\mathbf{Y}_f + \mathbf{Y}_p + \mathbf{Y}_l} = \frac{\mathbf{E}_g(\mathbf{Y}_f - g_m)}{\mathbf{Y}_f + \mathbf{Y}_p + \mathbf{Y}_l} \tag{5-69}$$

By combining these expressions, there results

$$\mathbf{E}_2 = \frac{\mathbf{Y}_f - g_m}{\mathbf{Y}_f + \mathbf{Y}_p + \mathbf{Y}_l} \frac{\mathbf{E}_1\mathbf{Y}_1 + \mathbf{E}_2\mathbf{Y}_f}{\mathbf{Y}_1 + \mathbf{Y}_g + \mathbf{Y}_f} \tag{5-70}$$

By solving for the gain, the result is

$$\mathbf{K}_f = \frac{\mathbf{E}_2}{\mathbf{E}_1} = \frac{\mathbf{Y}_1(\mathbf{Y}_f - g_m)}{(\mathbf{Y}_f + \mathbf{Y}_p + \mathbf{Y}_l)(\mathbf{Y}_1 + \mathbf{Y}_f + \mathbf{Y}_g) - \mathbf{Y}_f(\mathbf{Y}_f - g_m)}$$

which may be written in the form

$$\mathbf{K}_f = \frac{\mathbf{Y}_1(\mathbf{Y}_f - g_m)}{(\mathbf{Y}_1 + \mathbf{Y}_f + \mathbf{Y}_g)(\mathbf{Y}_p + \mathbf{Y}_l) + \mathbf{Y}_f(\mathbf{Y}_1 + \mathbf{Y}_g + g_m)} \tag{5-71}$$

To obtain an expression for the output impedance of this general feedback circuit, the procedure followed is substantially that used in Sec. 5-9. In the present case the equivalent circuit for this calculation becomes

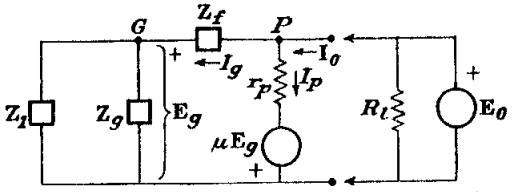


FIG. 5-34. The equivalent circuit for calculating the effective internal impedance of the feedback circuit of Fig. 5-33.

that shown in Fig. 5-34. The output admittance is given by

$$Y_{tf} = \frac{I_0}{E_0} = \frac{I_p + I_g}{E_0} \tag{5-72}$$

This may be written in the form

$$Y_{tf} = \frac{1}{E_0}\left[\frac{E_0 + \mu E_g}{r_p} + E_g(Y_g + Y_1)\right]$$

which is

$$Y_{tf} = \frac{1}{E_0}[(Y_1 + Y_g + g_m)E_g + Y_p E_0] \tag{5-73}$$

But from the diagram

$$E_g = \frac{Y_f}{Y_1 + Y_f + Y_g} E_0 \tag{5-74}$$

Hence the expression for the effective internal admittance becomes

$$Y_{tf} = Y_p + Y_f \frac{Y_1 + Y_g + g_m}{Y_1 + Y_f + Y_g} \tag{5-75}$$

An expression for the input admittance is readily obtained. It follows directly from Fig. 5-33*b* that

$$Y_{1f} \equiv \frac{I_i}{E_1} = \frac{E_1 - E_g}{Z_1 E_1} = Y_1\left(1 - \frac{E_g}{E_1}\right) \tag{5-76}$$

But by Eqs. (5-68) and (5-71)

$$E_g = \frac{E_1 Y_1 + K_f E_1 Y_f}{Y_1 + Y_g + Y_f} = E_1 \frac{Y_1 + K_f Y_f}{Y_1 + Y_g + Y_f} \tag{5-77}$$

Then

$$Y_{1f} = Y_1\left(1 - \frac{Y_1 + K_f Y_f}{Y_1 + Y_g + Y_f}\right)$$

which may be written in the form

$$Y_{1f} = \frac{Y_1}{Y_1 + Y_g + Y_f}[Y_g + (1 - K_f)Y_f] \tag{5-78}$$

A numerical comparison of the cathode follower, the anode follower, a conventional amplifier circuit under normal operating conditions, and a conventional amplifier with the plate resistance so chosen as to give the same gain as the cathode follower is very interesting. These data are given in Table 5-2, which assumes the use of a 6J5 tube, with $\mu = 20$, $r_p = 7{,}700$ ohms, R_l or $R_k = 10{,}000$ ohms, and $R_g = 1$ megohm in all cases.

TABLE 5-2

COMPARISON OF OPERATING FEATURES OF 6J5 TUBE IN VARIOUS CIRCUIT CONFIGURATIONS

Circuit	Gain	C_i, $\mu\mu$f.	R_i, megohms	Z_{tf}, ohms	Z_{0f}, ohms
1. Cathode follower	0.918	3.7	1	366	365
2. Anode follower ($Z_1 = Z_f = 0.5$ megohm)	− 0.818	9.6	0.538	853	785
3. Conventional amplifier	−11.3	45.2	1	7,700	4,350
4. Conventional amplifier (low gain)	− 0.918	9.9	1	7,700	353

It is first noted, by comparing the cathode follower with the anode follower, that the gain of each is roughly the same, and approximately equal to unity; the cathode-follower effective internal impedance and output-terminal impedances are approximately one-half those of the anode follower; the input impedance of the cathode follower is approximately twice that of the anode follower.

By comparing the cathode follower with the conventional grounded cathode circuit, the results bear out the statements made in Sec. 5-9, namely, that the gain is reduced; the effective input capacitance is less; the output-terminal impedance is less.

A comparison of the cathode follower with the conventional grounded-cathode circuit, but with the plate resistance chosen to yield numerically the cathode-follower gain, gives a somewhat better comparison between these two circuits. The input capacitance of the latter circuit is more than twice the former. However, this value of capacitance is not excessive. Also, the output-terminal impedance is nearly the same in both cases, although the effective internal impedance of the cathode follower is low, while that of the conventional amplifier circuit is high.

The question may then be raised whether there is any real advantage to be gained from using the cathode follower rather than a conventional amplifier with such a low plate resistance that the gain is intentionally made approximately equal to unity. The real advantage of the cathode follower and the anode follower over the conventional grounded-cathode

amplifier with adjusted gain lies principally in the advantages attendant to feedback amplifiers, as discussed in Sec. 5-2.

The cathode follower is extensively used. It is usually employed when a high input impedance or low output impedance or both are required. For example, it is used as the input stage to almost all good-quality cathode-ray oscilloscopes. It is used when it is required to transmit a signal over a relatively long distance. The low output-terminal impedance of the cathode follower tends to minimize the capacitive loading of the long wires or shielded cable, i.e., a low RC time constant results, and transient signals are not seriously distorted. Moreover, with the proper choice of tube, an output-terminal impedance which provides a good impedance match with the characteristic impedance of a coaxial cable is possible.

It is worth noting that for the case of large amplitude signals with an appreciable load capacitance neither the cathode follower nor the anode follower provides the same positive and negative response characteristics. The cathode follower provides a faster positive- than a negative-output change, whereas the anode follower provides a faster negative- than a positive-output change. The reasons for this action are to be found in the fact that the circuits cannot be treated accurately as linear circuits for large signal amplitudes. The replacement of the single tube by a series-connected push-pull pair overcomes this effect. Such a circuit is given[6] in Fig. 5-37.

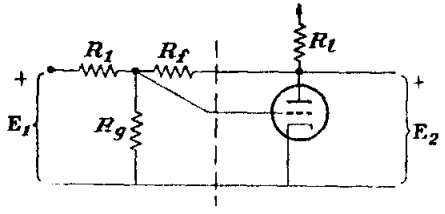


FIG. 5-35. The anode follower drawn in a manner to emphasize the summing-circuit character of the amplifier.

An interesting physical explanation of the operation of the anode follower is possible,[6] and this discussion is examined analytically in some detail in Sec. 8-2 in connection with summing circuits and in Sec. 8-3 as an approximate analysis of the general operational feedback amplifier. If R_1, R_g, and the tube circuit are considered as elements of a summing network and the circuit of the anode follower is drawn in Fig. 5-35 to stress this summing-network character, then R_g acts as a comparator, the potential across it being the difference between the input and the output potentials. The action of the amplifier is to tend to make the difference zero, higher amplifier gain being accompanied by a smaller difference, or closer equality. In the extreme case when $R_1 = R_f$, and with very high amplifier gain, the difference between the input and output potentials is zero, the resultant system gain being unity. The increased gain may be included in the output stage (by using the high-gain tube of Fig 5-35), or it may be included to provide a more selective comparator. A circuit in which the difference signal is compared at the anode of the comparator stage after amplification is given in Fig. 5-36. Considerable improvement is effected in this circuit.

Another fact to be noted from Eqs. (5-75) and (5-78) is that an increase in $\mathbf{Y}_f$ (a reduction in $\mathbf{Z}_f$) results in an increase in the effective input impedance, and a decrease of the effective output-terminal impedance. The substitution of a tube impedance for the shunt resistance R_f of the feedback path improves these characteristics. A circuit of this type is given in Fig. 5-37 and also in Prob. 5-34.

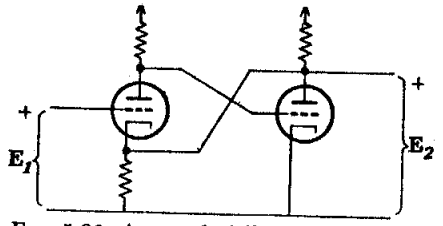


FIG. 5-36. An anode-follower circuit with an amplifier comparator.

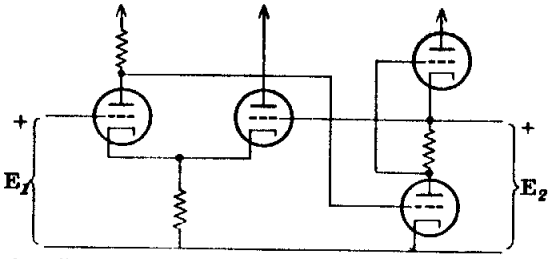


FIG. 5-37. A push-pull-connected anode follower with amplifier comparator and tube feedback path.

5-12. The Cathode-follower Amplifier. Owing to the high input impedance and low output-terminal impedance of the cathode follower, it is anticipated that the use of such a device as the coupling amplifier between stages will provide an amplifier with a broad frequency-response range. This matter is examined analytically.

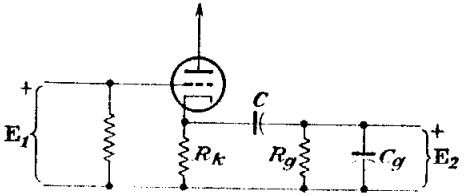


FIG. 5-38. An RC coupled cathode-follower stage.

When used as a coupling amplifier between stages, the cathode-follower circuit becomes essentially that illustrated in Fig. 5-38. In this circuit, C_g is the sum of the effective output capacitance of the cathode follower and the effective input capacitance of the next stage.

The gain of this amplifier will be examined for the various frequency ranges. These follow directly from Eq. (5-54) with the proper interpretation of $\mathbf{Z}_k$.

Mid-frequency Gain. The interelectrode capacitances are negligible over the mid-frequency band, whence C_g may be neglected. Also the

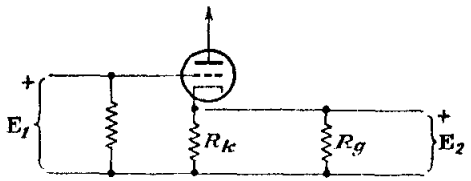


FIG. 5-39. The mid-frequency cathode-follower circuit.

coupling capacitor C is assumed sufficiently large so that its reactance is negligible. The resulting circuit becomes that of Fig. 5-39. The mid-frequency gain becomes

$$\mathbf{K}_0 = \frac{\mu \mathbf{Z}_k'}{r_p + (\mu + 1)\mathbf{Z}_k'} = \frac{\mu}{\mu + 1 + r_p/\mathbf{Z}_k'} \tag{5-79}$$

where

$$\frac{1}{\mathbf{Z}_k'} = \frac{1}{R_k} + \frac{1}{R_g} \tag{5-80}$$

H-F Gain. At the h-f end of the response curve, the coupling capacitor may be omitted, although the effect of C_g becomes important. The

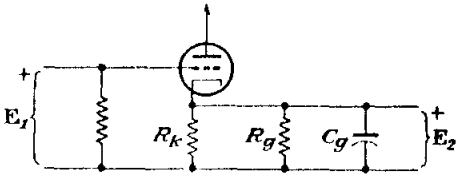


FIG. 5-40. The h-f circuit of the cathode-follower amplifier.

equivalent circuit has the form shown in Fig. 5-40. The gain equation [Eq. (5-54)] now becomes

$$\mathbf{K}_2 = \frac{\mu}{\mu + 1 + r_p/\mathbf{Z}_k''} \tag{5-81}$$

where

$$\frac{1}{\mathbf{Z}_k''} = \frac{1}{R_k} + \frac{1}{R_g} + j\omega C_g \tag{5-82}$$

The gain ratio $\mathbf{K}_2/\mathbf{K}_0$ becomes

$$\frac{\mathbf{K}_2}{\mathbf{K}_0} = \frac{1}{1 + \dfrac{j\omega C_g r_p}{\mu + 1 + r_p/\mathbf{Z}_k'}}$$

which is

$$\frac{\mathbf{K}_2}{\mathbf{K}_0} = \frac{1}{1 + j\omega r_p' C_g} = \frac{1}{1 + j(f/f_2)} \tag{5-83}$$

where

$$r_p' = \frac{r_p}{\mu + 1 + r_p/\mathbf{Z}_k'} \qquad f_2 = \frac{1}{2\pi r_p' C_g} \tag{5-84}$$

It should be noted that this expression has substantially the same form as Eq. (4-17) for the conventional RC circuit. However, since the product $r_p' C_g$ for the cathode follower is much smaller than $r_p C_g$ of the RC amplifier, then the upper frequency limit of uniform response is much greater for the cathode follower than for the conventional RC stage. Because of this, it is possible to achieve a high h-f limit even when the

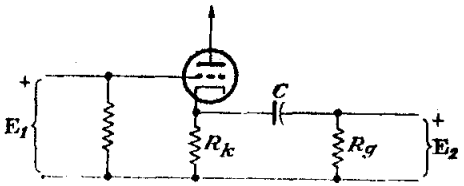


FIG. 5-41. The l-f circuit of the cathode-follower amplifier.

cathode follower is followed by a stage having a high input capacitance. This means that a rather wide frequency response is possible under these conditions even with a following triode stage.

L-F Gain. At the low frequencies, C_g may be neglected, and the effect of the coupling capacitor becomes very important. Equation (5-54) appropriately modified becomes

$$\mathbf{K}_1 = \frac{\mu}{\mu + 1 + r_p/\mathbf{Z}_k'''} \frac{R_g}{R_g + 1/j\omega C} \tag{5-85}$$

This expression may be written in the form

$$\mathbf{K}_1 = \frac{\mu}{\mu + 1 + \dfrac{r_p}{\mathbf{Z}_k'} - \dfrac{r_p}{R_g(1 + j\omega C R_g)}} \frac{j\omega C R_g}{1 + j\omega C R_g} \tag{5-86}$$

where use has been made of the fact that

$$\frac{1}{\mathbf{Z}_k'''} = \frac{1}{R_k} + \frac{1}{R_g + 1/j\omega C} = \frac{1}{\mathbf{Z}_k'} - \frac{1}{R_g(1 + j\omega C R_g)} \tag{5-87}$$

The gain ratio becomes

$$\frac{\mathbf{K}_1}{\mathbf{K}_0} = \frac{1}{\mu + 1 + \dfrac{r_p}{\mathbf{Z}_k'} - \dfrac{r_p}{R_g(1 + j\omega C R_g)}} \frac{j\omega C R_g}{1 + j\omega C R_g} \left(\mu + 1 + \frac{r_p}{\mathbf{Z}_k'}\right)$$

which reduces to the form

$$\frac{\mathbf{K}_1}{\mathbf{K}_0} = \frac{1}{1 - j(1/\omega C \mathbf{R}_1)} = \frac{1}{1 - j(\mathbf{f}_1/f)}$$

where

$$\mathbf{R}_1 = \frac{R_g}{1 - \dfrac{r_p/R_g}{\mu + 1 + r_p/\mathbf{Z}_k'}} \qquad \mathbf{f}_1 = \frac{1}{2\pi C \mathbf{R}_1} \tag{5-88}$$

This expression has substantially the same form as Eq. (4-11) for the l-f gain of the *RC*-coupled amplifier, and under typical operating conditions the value of f_1 is much the same for the cathode follower as for the *RC* amplifier.