TUNED POTENTIAL AMPLIFIERS

Tuned potential amplifiers are used in those cases in which it is desired to amplify a relatively narrow band of frequencies centered about some designated mean or carrier frequency. Potentials whose frequencies lie outside of this range are undesirable and are to be rejected. The use of tuned networks accomplishes this, as it is possible to adjust the tuned network so that the impedance falls steeply to low values outside of the

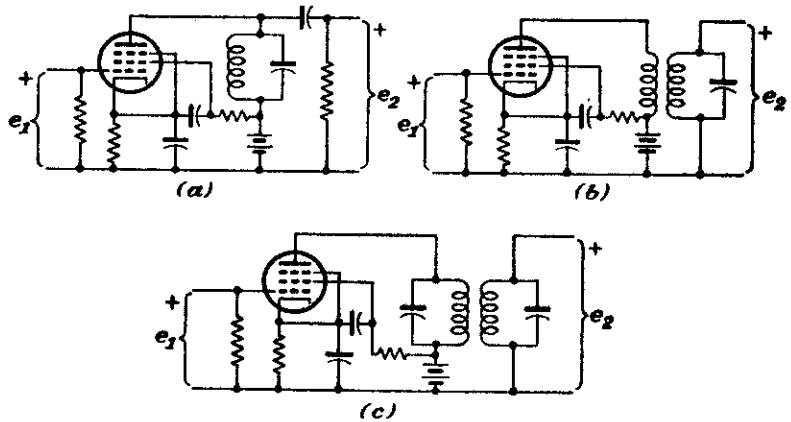


FIG. 11-1. The three basic tuned amplifier circuits: (*a*) single-tuned, direct coupling; (*b*) single-tuned, transformer coupling; (*c*) double-tuned.

desired frequency band, with the consequent reduction in amplifier gain to negligibly low values. The resulting nonlinear distortion that is produced in these amplifiers is very small, both because the stage is operated under class A conditions and because the tuned plate-circuit impedance may be very low for any harmonic frequencies that might be generated within the tube.

There are three basic amplifier circuits, and these are illustrated in Fig. 11-1. Pentodes are ordinarily used in such amplifiers, and the circuits are drawn showing such tubes. In two of these types, a single resonant circuit is used, which may be included directly in the plate circuit (direct-coupled) or which may be inductively coupled to the plate circuit (transformer-coupled). In the third type, a double-tuned band-pass arrangement is used, both the primary and secondary circuits being tuned. An analysis of the operation of each of these amplifier circuits will be given.

11-1. Single-tuned Direct-coupled Amplifier. The equivalent circuit of a typical single-tuned direct-coupled stage is given in Fig. 11-2. Included in this diagram are the output tube capacitances [see Eq. (3-23)] and the input and wiring capacitances to the following stage. The coupling capacitance may be neglected, as this is presumed to be large.

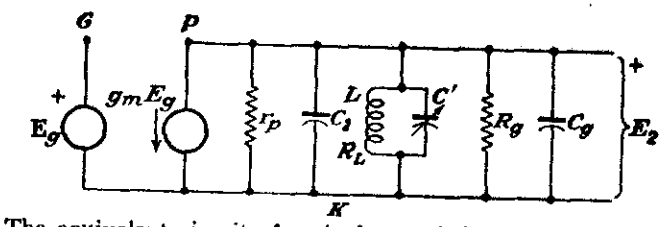


FIG. 11-2. The equivalent circuit of a single-tuned direct-coupled class A amplifier.

In accordance with the discussion of Sec. 3-7, the gain of the amplifier can be written directly as

$$\mathbf{K} = -g_m \mathbf{Z} \tag{11-1}$$

where $\mathbf{Z}$ is the total load impedance. This impedance has the complex form given by

$$\frac{1}{\mathbf{Z}} = \frac{1}{r_p} + \frac{1}{R_g} + \frac{1}{\mathbf{Z}_t} \tag{11-2}$$

where $\mathbf{Z}_t$ is the impedance of the antiresonant circuit and comprises the inductance L and the sum of the various capacitances $C_t = C_2 + C_g + C'$, where C_2 is defined as in Eq. (3-23).

The impedance $\mathbf{Z}_t$ has the form

$$\mathbf{Z}_t = \frac{-j\frac{1}{\omega C}(R_L + j\omega L)}{R_L + j\left(\omega L - \frac{1}{\omega C}\right)} \tag{11-3}$$

which may be written in the form

$$\mathbf{Z}_t = \frac{\frac{L}{C}\left(1 - j\frac{R_L}{\omega L}\right)}{R_L\left[1 + j\frac{\omega L}{R_L}\left(1 - \frac{1}{\omega^2 LC}\right)\right]} \tag{11-4}$$

By writing

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\delta = \frac{\omega}{\omega_0} - 1 \tag{11-5}$$

$$Q = \frac{\omega_0 L}{R_L} = \frac{1}{\omega_0 C R_L} = \frac{1}{R_L}\sqrt{\frac{L}{C}}$$

the impedance function becomes

$$\mathbf{Z}_t = \frac{R_L Q^2 \left(1 - j\,\frac{1}{Q}\,\frac{\omega_0}{\omega}\right)}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

$$\mathbf{Z}_t = \frac{R_L Q^2 \left[1 + \delta - j\,\frac{1}{Q}\right]}{1 + \delta + jQ\delta(2 + \delta)} \tag{11-6}$$

At resonance $\omega = \omega_0$, and $\delta = 0$. Then

$$\mathbf{Z}_t = R_L Q^2 \left(1 - j\,\frac{1}{Q}\right) \tag{11-7}$$

Since Q for the circuit used is usually high, with $Q \geqq 10$, then with good approximation

$$\mathbf{Z}_t = R_0 \doteq R_L Q^2 \tag{11-8}$$

This result shows that the shunt impedance R_0 of the antiresonant circuit for circuits with $Q > 10$ is essentially resistive at the resonant frequency.

By combining Eq. (11-8) with Eq. (11-2), the gain at resonance becomes

$$\mathbf{K}_{res} = \frac{-g_m}{\frac{1}{r_p} + \frac{1}{R_g} + \frac{1}{R_L Q^2}} = \frac{-g_m \omega_0 L Q}{1 + \frac{\omega_0 L Q}{r_p} + \frac{\omega_0 L Q}{R_g}} \tag{11-9}$$

This expression may be written in the form

$$\mathbf{K}_{res} = -g_m \omega_0 L Q_e \tag{11-10}$$

where Q_e, the effective Q of the amplifier, is

$$Q_e = \frac{Q}{1 + \omega_0 L Q/r_p + \omega_0 L Q/R_g} \tag{11-11}$$

This is the equivalent Q of the resonance curve of the tuned amplifier and is the Q of the actual resonant circuit as modified by the shunting resistances R_g and r_p.

To find the gain of the amplifier when the input frequency and the resonant frequency of the tuned circuit are slightly different from each other, it is supposed that $\omega \doteq \omega_0$, whence δ is small. From Eq. (11-6), it follows that

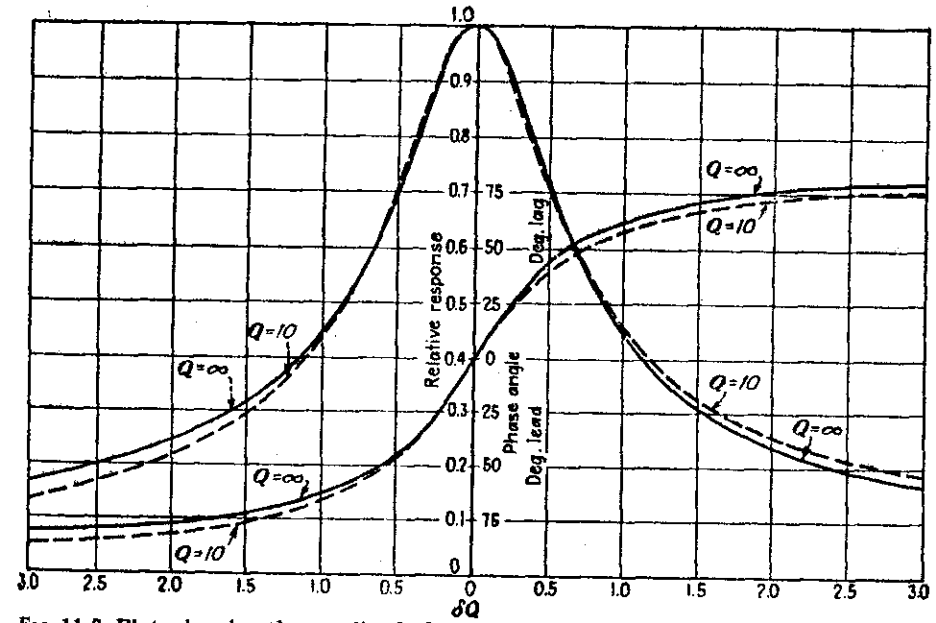


FIG. 11-3. Plots showing the amplitude K/K_{res} and phase of the output of a single-tuned direct-coupled amplifier. (*From F. E. Terman, "Radio Engineering," 3d ed., McGraw-Hill Book Company, Inc., New York*, 1947.)

$$\mathbf{Z}_t = R_L Q^2 \frac{1 - j(1/Q)}{1 + 2j\delta Q} \doteq \frac{R_L Q^2}{1 + 2j\delta Q} \tag{11-12}$$

The corresponding value of the gain, given by Eq. (11-9), is

$$\mathbf{K} = -\frac{g_m}{\frac{1}{r_p} + \frac{1}{R_g} + \frac{1 + 2j\delta Q}{R_L Q^2}} \tag{11-13}$$

The gain ratio $\mathbf{K}/\mathbf{K}_{res}$ is then

$$\frac{\mathbf{K}}{\mathbf{K}_{res}} = \frac{1}{1 + j2\delta Q_e} \tag{11-14}$$

from which the amplitude ratio is

$$\frac{K}{K_{res}} = \frac{1}{\sqrt{1 + (2\delta Q_e)^2}} \tag{11-15}$$

A plot of these results is given in Fig. 11-3. This is essentially the "universal resonance" curve. Note from Eq. (11-15) that when

$$2\delta Q_e = 1$$

then

$$\frac{K}{K_{res}} = \frac{1}{\sqrt{2}}$$

But since the bandwidth of the circuit is the frequency width between the 3-db power points, then for a symmetrical gain characteristic

$$B = 2(f_{3db} - f_0) = \frac{2(f_{3db} - f_0)f_0}{f_0}$$

or

$$B = 2\delta_{3db}f_0 = \frac{f_0}{Q_e} \tag{11-16}$$

In order that the potential gain at resonance be large, the resonant impedance of the tuned circuit, and the grid resistor R_g, must be large. It might appear from Eq. (11-10) that higher gains are accomplished by choosing large values of L/C. Note, however, from Eqs. (11-5) and (11-8) that an increase of $\mathbf{Z}_t$ at resonance by increasing the L/C ratio is accompanied by a decreased Q_e, with a corresponding increase of bandwidth or decreased frequency selectivity. This is an undesirable condition if a narrow bandwidth is desired, but it is an important consideration in wide-band tuned amplifiers. Note also that if the circuit Q of the tuned circuit is increased at fixed values of ω_0 and L/C ratio, then the effective Q_e of the amplifier is increased, with a corresponding increase of frequency selectivity or decreased bandwidth.

11-2. Single-tuned Transformer-coupled Amplifier. The general behavior of the single-tuned transformer-coupled amplifier is quite similar to that of the direct-coupled circuit. To examine the operation of the circuit in some detail, refer to Fig. 11-4, which gives the equivalent circuit of this amplifier. An approximate expression for the potential gain of the amplifier is readily obtained. If it is noted that r_p is usually large compared with R_1 and ωL_1, then the potential induced in the secondary of the transformer is given by

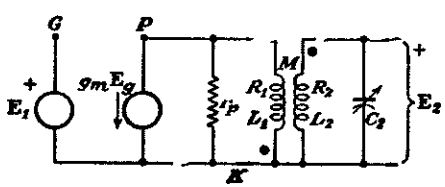


FIG. 11-4. The equivalent circuit of the single-tuned transformer-coupled potential amplifier.

$$\mathbf{E}_{ind} \doteq j\omega M(g_m\mathbf{E}_g) = j\omega M(g_m\mathbf{E}_1) \tag{11-17}$$

since in this circuit $\mathbf{E}_g = \mathbf{E}_1$. The output potential, which is the potential across the capacitor C_2, is then

$$\mathbf{E}_2 = \frac{j\omega M(g_m\mathbf{E}_1)}{R_2 + j(\omega L_2 - 1/\omega C_2) + \omega^2M^2/r_p} \frac{1}{j\omega C_2} \tag{11-18}$$

where ω^2M^2/r_p is the reflected impedance of the primary into the secondary circuit. The expression for the gain then becomes

$$\mathbf{K} = \frac{\mathbf{E}_2}{\mathbf{E}_1} = \frac{\mu M/C_2}{r_p[R_2 + j(\omega L_2 - 1/\omega C_2)] + \omega^2M^2} \tag{11-19}$$

The corresponding expression for the potential gain at resonance is

$$\mathbf{K}_{res} = \frac{\mu M/C_2}{r_pR_2 + \omega_0^2M^2} \tag{11-20}$$

This result may also be expressed in terms of $Q_2 = \omega_0L_2/R_2 = 1/\omega_0C_2R_2$ and is

$$\mathbf{K}_{res} = g_m \frac{\omega_0MQ_2}{1 + \omega_0^2M^2/r_pR_2} \tag{11-21}$$

which may be written as

$$\mathbf{K}_{res} = g_m\omega_0MQ_e \tag{11-22}$$

where the effective value Q_e is

$$Q_e = \frac{Q_2}{1 + \omega_0^2M^2/r_pR_2} \tag{11-23}$$

A comparison of this expression with Eq. (11-10) shows that transformer coupling modifies the amplification by the ratio M/L. This provides a means for controlling the gain of the stage and still retaining the high Q required for selectivity. It might appear that there are no limits on the gain and that it continues to increase with increasing values of M. This is not so, owing to the appearance of M in the denominator of Eq. (11-20). An optimum value of gain exists, and this occurs when M has the value required to make $\partial K_{res}/\partial M = 0$. This yields, for the optimum value of M,

$$\frac{\partial K_{res}}{\partial M} = \frac{\mu/C_2}{r_pR_2 + \omega_0^2M^2} - \frac{(\mu M/C_2)2\omega_0^2M}{(r_pR_2 + \omega_0^2M^2)^2} = 0$$

or

$$M_{opt} = \frac{\sqrt{r_pR_2}}{\omega_0} \tag{11-24}$$

Equation (11-21) becomes

$$\mathbf{K}_{res,opt} = g_m \frac{\sqrt{r_pR_2}\,Q_2}{2} = \frac{\mu Q_2}{2}\sqrt{\frac{R_2}{r_p}} \tag{11-25}$$

To find the bandwidth of the amplifier, consider the general expression for the gain given by Eq. (11-19). By writing, as before,

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\delta = \frac{\omega}{\omega_0} - 1 \tag{11-26}$$

$$Q_2 = \frac{\omega_0 L_2}{R_2} = \frac{1}{\omega_0 C_2 R_2} = \frac{1}{R_2}\sqrt{\frac{L_2}{C_2}}$$

and noting that in the neighborhood of resonance

$$\omega L_2 - \frac{1}{\omega C_2} = \sqrt{\frac{L_2}{C_2}}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

$$\doteq 2\delta\sqrt{\frac{L_2}{C_2}} = 2\delta R_2 Q_2 \tag{11-27}$$

Eq. (11-19) becomes

$$\mathbf{K} = \frac{\mu M/C_2}{r_p R_2\left(1 + j\frac{1}{R_2}2\delta\sqrt{\frac{L_2}{C_2}}\right) + \omega^2 M^2} \tag{11-28}$$

The gain ratio [Eqs. (11-28) to (11-20)] then becomes

$$\frac{\mathbf{K}}{\mathbf{K}_{res}} = \frac{1}{1 + j2\delta Q_e} \tag{11-29}$$

which has the same form as for the direct-coupled connection [Eq. (11-14)]. The bandwidth of this amplifier is, following the same reasoning as that which led to Eq. (11-16),

$$B = \frac{f_0}{Q_e} \tag{11-30}$$

The optimum value of M is not of much importance, owing to practical limitations. This follows from Eq. (11-24), which shows that for pentodes, with the corresponding large values of r_p, the value of M would be large. In fact, to achieve these values of M, the distributed capacitances of the windings may become excessive, and the self-resonant frequency may be so low as to make the coils useless. Owing to this, the mutual inductance is usually chosen far below the optimum value in the pentode amplifier.

11-3. The Double-tuned Amplifier. Both the single-tuned direct-coupled amplifier and the double-tuned amplifier are extensively used in radar, television, and communication receivers. For the i-f amplifiers of both a-m and f-m types, the double-tuned amplifier is commonly used. This is so because such an amplifier can provide substantially constant amplification over a band of frequencies and the gain falls more sharply outside of this band than does the single-tuned stage.

To examine the operation of the circuit, refer to the equivalent circuit of the amplifier given in Fig. 11-5. This circuit can be further simplified by applying Thévenin's theorem to the portion of the circuit to the left of the points *aa*. The equivalent generator has the potential

$$\mathbf{E} = \frac{\mu\mathbf{E}_1(1/j\omega C_1)}{r_p + 1/j\omega C_1} \tag{11-31}$$

where $C_1 = C_p' + C_p''$. But since $r_p > 1/\omega C_1$ for the pentode, then with good approximation

$$\mathbf{E} \doteq \frac{\mu\mathbf{E}_1}{j\omega r_p C_1} = \frac{g_m\mathbf{E}_1}{j\omega C_1} \tag{11-32}$$

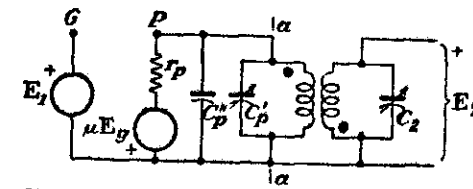


FIG. 11-5. The equivalent circuit of a double-tuned amplifier.

The internal impedance of the equivalent generator will have the value

$$\mathbf{Z} = \frac{r_p(1/j\omega C_1)}{r_p + 1/j\omega C_1}$$

which is, to the same approximation as above,

$$\mathbf{Z} \doteq \frac{1}{j\omega C_1} \tag{11-33}$$

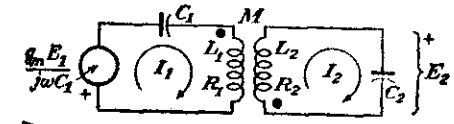


FIG. 11-6. The equivalent series form of Fig. 11-5.

Then the equivalent circuit of Fig. 11-5 reduces to the form of Fig. 11-6.

This circuit is analyzed by the standard methods of network analysis. Accordingly, if one writes

$$\mathbf{E} = \varrho_{11}\mathbf{I}_1 + \varrho_{12}\mathbf{I}_2$$
$$0 = \varrho_{12}\mathbf{I}_1 + \varrho_{22}\mathbf{I}_2 \tag{11-34}$$

then the current in the secondary is

$$\mathbf{I}_2 = -\frac{\mathbf{E}\varrho_{12}}{\varrho_{11}\varrho_{22} - \varrho_{12}^2} \tag{11-35}$$

where

$$\varrho_{11} = R_1 + j\left(\omega L_1 - \frac{1}{\omega C_1}\right)$$
$$\varrho_{12} = j\omega M \tag{11-36}$$
$$\varrho_{22} = R_2 + j\left(\omega L_2 - \frac{1}{\omega C_2}\right)$$

The gain of the amplifier becomes

$$\mathbf{K} = \frac{\mathbf{E}_2}{\mathbf{E}_1} = \frac{(g_m/j\omega C_1)(1/j\omega C_2)j\omega M}{\varrho_{11}\varrho_{22} - \varrho_{12}^2} \tag{11-37}$$

But it must be noted that both circuits are tuned to the same resonant

frequency. Thus

$$\omega_0 = \frac{1}{\sqrt{L_1C_1}} = \frac{1}{\sqrt{L_2C_2}}$$

Also define*

$$Q_1 = \frac{\omega_0 L_1}{R_1} = \frac{1}{\omega_0 C_1 R_1}$$
$$Q_2 = \frac{\omega_0 L_2}{R_2} = \frac{1}{\omega_0 C_2 R_2} \tag{11-38}$$

and write

$$a = \frac{\omega_0 M}{\sqrt{R_1 R_2}} = k\sqrt{Q_1 Q_2}$$

Then, by Eq. (11-27),

$$\varrho_{11} = R_1 + j\left(\omega L_1 - \frac{1}{\omega C_1}\right) = R_1(1 + j2\delta Q_1)$$

Similarly $\varrho_{22} = R_2(1 + j2\delta Q_2)$ (11-39)

and $\varrho_{12} = j\omega M$

The expression for the gain [Eq. (11-37)] then becomes for frequencies near resonance

$$\mathbf{K} = \frac{-jg_m(M/\omega_0 C_1 C_2)}{R_1 R_2(1 + j2\delta Q_1)(1 + j2\delta Q_2) + \omega_0^2 M^2}$$

$$\mathbf{K} = \frac{-jg_m \dfrac{\omega_0 M}{\sqrt{R_1 R_2}} Q_1 Q_2 \sqrt{R_1 R_2}}{1 + \omega_0^2 M^2/R_1 R_2 + 2j\delta(Q_1 + Q_2) - 4\delta^2 Q_1 Q_2}$$

or finally

$$\mathbf{K} = \frac{-jag_m Q_1 Q_2 \sqrt{R_1 R_2}}{1 + a^2 + j2\delta(Q_1 + Q_2) - 4\delta^2 Q_1 Q_2} \tag{11-40}$$

The gain at resonance is obtained by setting $\delta = 0$ in this expression. There results

$$\mathbf{K}_{res} = \frac{-jag_m Q_1 Q_2 \sqrt{R_1 R_2}}{1 + a^2} \tag{11-41}$$

The gain ratio at frequencies slightly different from resonance is given by

$$\frac{\mathbf{K}}{\mathbf{K}_{res}} = \frac{1}{\left(1 - \dfrac{4\delta^2 Q_1 Q_2}{1 + a^2}\right) + j\dfrac{2\delta(Q_1 + Q_2)}{1 + a^2}} \tag{11-42}$$

The exact shape of the response curve of the double-tuned system depends upon the parameter a, or, correspondingly, on k, the coefficient of coupling between the primary and secondary coils. The resonant gain is a maximum when $a = 1$, as may be verified by examining $d\mathbf{K}/da = 0$. Moreover, if the primary and secondary Q values are the same and $a = 1$, the response curve has the maximum possible single-peak flatness in the vicinity of resonance. This is the condition for critical coupling k_c. A value of a greater than unity (overcoupling) results in double peaks, whereas a value of a less than unity (undercoupling) causes the response to be rounded on the top. If the circuit is considerably undercoupled, then the gain may be less than that at resonance. The situation discussed is illustrated graphically in Fig. 11-7.

An analytic expression for the positions of the peaks in the output of the overcoupled circuit is readily possible. These are given, of course, by the values of frequency at which Eq. (11-40) is a maximum. To find these

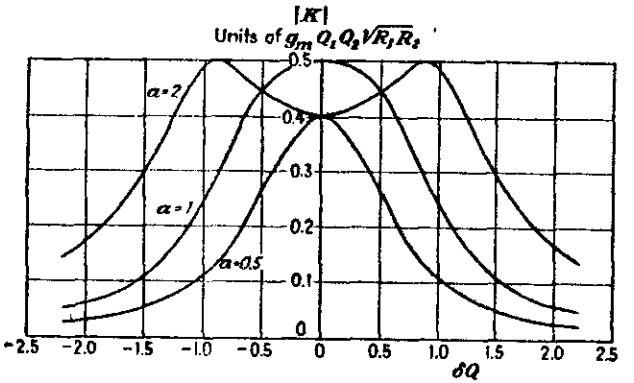


FIG. 11-7. The response characteristics of a double-tuned amplifier for various values of coupling.

values, it is noted that the gain is a maximum without regard to the phase. Thus the square of the absolute value of Eq. (11-40) is differentiated with respect to δ and maximized. The results are

$$|K|^2 = (g_m Q_1 Q_2 \sqrt{R_1 R_2})^2 \frac{a^2}{(1 + a^2 - 4\delta^2 Q_1 Q_2)^2 + [2\delta(Q_1 + Q_2)]^2} \tag{11-43}$$

and the derivative $\partial|K|^2/\partial\delta = 0$ yields

$$1 + a^2 - 4\delta^2 Q_1 Q_2 = \frac{(Q_1 + Q_2)^2}{2Q_1 Q_2}$$

from which

$$\delta = \pm\frac{1}{2}\sqrt{k^2 + \frac{1}{Q_1 Q_2} - \frac{1}{2}\left(\frac{Q_1 + Q_2}{Q_1 Q_2}\right)^2} \tag{11-44}$$

Frequently the circuits are designed with $Q_1 = Q_2$. Even if this condition is not true, ordinarily Q_1 does not differ too markedly from Q_2, and it is possible to assume that

$$\sqrt{Q_1 Q_2} \doteq \frac{Q_1 + Q_2}{2}$$

* If the amplifier is one of a chain, the loading effect of the following stage should be included in Q_2.

Subject to this approximation, Eq. (11-44) becomes

$$\delta = \pm \frac{1}{2}\sqrt{k^2 - \frac{1}{Q_1 Q_2}} \tag{11-45}$$

which becomes, in the manner of representation of Fig. 11-7, simply

$$\delta \sqrt{Q_1 Q_2} = \pm \tfrac{1}{2}\sqrt{a^2 - 1} \tag{11-46}$$

The value of the gain at either peak K_{max} is obtained by combining Eq. (11-46) with (11-40). The result is

$$\mathbf{K}_{max} = -j(g_m Q_1 Q_2 \sqrt{R_1 R_2}) \frac{a}{2(1 + j\sqrt{a^2 - 1})}$$

or

$$K_{max} = \tfrac{1}{2}(g_m Q_1 Q_2 \sqrt{R_1 R_2}) \tag{11-47}$$

This shows that for the overcoupled-case the maximum gain is the same as that for critical coupling $a = 1$, and at resonance $\delta = 0$.

The gain at the dip, at the frequency ω_0, can be found readily by setting $\delta = 0$ in Eq. (11-40). The result is

$$\mathbf{K}_{min} = -j(g_m Q_1 Q_2 \sqrt{R_1 R_2}) \frac{a}{1 + a^2}$$

or

$$K_{min} = g_m Q_1 Q_2 \sqrt{R_1 R_2} \frac{a}{1 + a^2} \tag{11-48}$$

It follows from Eqs. (11-46) and (11-47) that increased coupling increases the frequency separation of two peaks but does not change their amplitudes. If the coupling is very large, then the approximation made in Eq. (11-40) is no longer valid. The effect of the factor ω/ω_0 in this equation is to increase the lower frequency maximum and decrease the higher frequency maximum.

The corresponding dependence of the current in the primary winding on the frequency is of some interest. It is obtained directly by solving Eq. (11-34) for $\mathbf{I}_1$. The results are illustrated in Fig. 11-8. For the case when the coefficient of coupling k is small, the secondary circuit has little effect in the primary, and the resulting response is essentially that of the simple resonant circuit. When $k = k_c$, the reflected resistance from the secondary decreases the current at resonance. On each side of resonance, the reflected reactance is either inductive (below resonance) or capacitive (above resonance). A point is reached on each side of resonance when the reflected reactances cancel the primary reactances, which are capacitive below resonance and inductive above resonance. As a result, the current shows peaks. For values $k > k_c$, the double peak in the primary circuit becomes more pronounced.

For the case where the primary and secondary Q values are not the same, Aiken[1] has shown that a somewhat higher value of k is needed to produce double peaks in the secondary. The flattest selectivity curve may be shown to occur when

$$a^2 = \frac{1}{2}\left(\frac{Q_1}{Q_2} + \frac{Q_2}{Q_1}\right) \tag{11-49}$$

although the mid-band gain is not a maximum under these conditions. When Eq. (11-49) is satisfied, the circuit is said to be *transitionally* coupled. The transitional value of coupling coefficient is, by Eqs. (11-49) and (11-38),

$$k_t = \sqrt{\frac{1}{2}\left(\frac{1}{Q_1^2} + \frac{1}{Q_2^2}\right)} \tag{11-50}$$

For a coupling coefficient larger than this value, the selectivity curve divides into two peaks. For values less than this, the curve has a single peak.

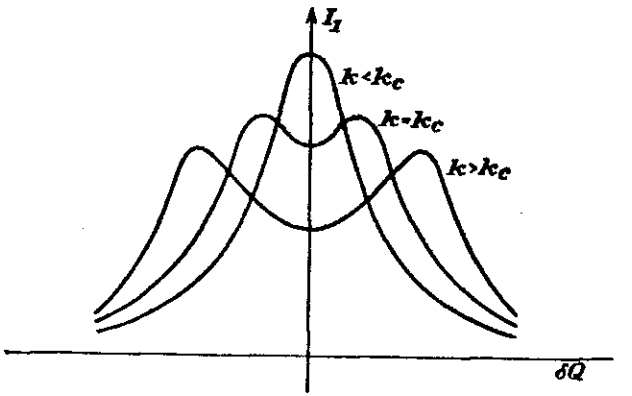


FIG. 11-8. The primary current of a double-tuned amplifier, corresponding to the conditions of Fig. 11-7.

For different primary and secondary Q values, but with $L_1 = L_2 = L$, and for k greater than the transitional coupling coefficient k_t, Aiken has shown that the frequency spread between peaks is

$$\Delta\omega = \frac{1}{L}\sqrt{(\omega_0 M)^2 - \frac{R_1^2 + R_2^2}{2}} \tag{11-51}$$

If $R_1 = R_2 = R$, this becomes

$$\Delta\omega = \frac{1}{L}\sqrt{(\omega_0 M)^2 - R^2}$$

from which it follows that

$$\frac{\Delta\omega}{\omega_0} = \frac{1}{\omega_0 L}\sqrt{(\omega_0 M)^2 - R^2} = \sqrt{k^2 - k_c^2} \tag{11-52}$$

The dip at resonance when k exceeds k_t is a function of the ratio k/k_c. Aiken shows this relation to have the form illustrated in Fig. 11-9. The bandwidth over which the response remains above the center value is $\sqrt{2}\,\Delta f$.

It is interesting to compare the gain of the double-tuned circuit with a single-tuned circuit having the same Q. The gains of the two circuits at resonance are given by Eqs. (11-10), and (11-41) for optimum value of a, and are

Single-tuned direct-coupled: $\mathbf{K}_{res} = -g_m\omega_0 LQ_e$

Double tuned (with $a = 1$ and identical coils):

$$\mathbf{K}_{res} = -j0.5g_mQ^2R = -j0.5g_m\omega_0 LQ$$

It is observed that for critical coupling the gains at resonance of the two amplifiers are identical if it is assumed that the tuning capacitance of the

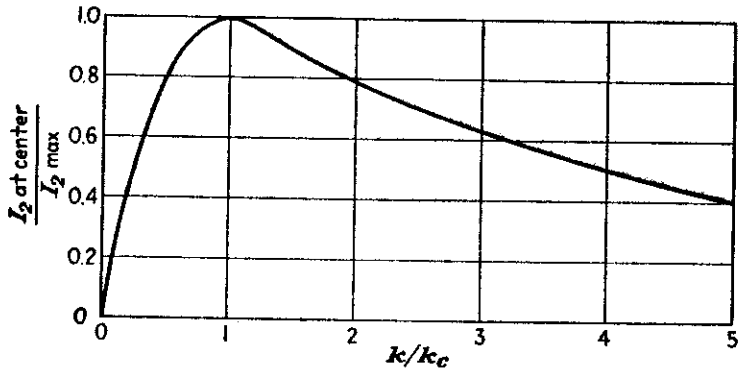


FIG. 11-9. The effect of the ratio of actual to critical coupling in a double-tuned circuit.

individual tuned circuit in the double-tuned circuit is one-half the tuning capacitance for the single-tuned case.

Despite the fact that the response characteristics are optimum under critical coupling conditions, the transformers in narrow-band double-tuned amplifiers are usually undercoupled slightly. This is done in order that the frequency alignment of the tuned circuits may be made easier, since, with undercoupled stages, each stage can be adjusted separately to give maximum response at the specified frequency. If overcoupled circuits exist, owing to the interactions between coils and the resulting double peak, this alignment is more critical. The critical coupling case is likewise difficult to align.

The bandwidth of the amplifier, under optimum conditions $a = 1$ and with equal primary and secondary values of Q, is readily calculated. Under these conditions Eq. (11-42) becomes

$$\frac{\mathbf{K}}{\mathbf{K}_{res}} = \frac{1}{1 - 2\delta^2Q^2 + j2\delta Q} \tag{11-53}$$

and the magnitude becomes

$$\frac{K}{K_{res}} = \frac{1}{\sqrt{1 + 4\delta^4Q^4}} \tag{11-54}$$

Since, by definition, the bandwidth gives a measure of the frequency spread over which the gain remains within 3 db of the maximum value, then

$$4\delta^4Q^4 = 1$$

from which it follows that the bandwidth is

$$B = 2\delta f_0 = \sqrt{2}\frac{f_0}{Q} \tag{11-55}$$

A comparison of this result with Eq. (11-16) for the single-tuned stage shows that the 3-db bandwidth of the double-tuned circuit is 1.414 times that of the single-tuned stage.

11-4. Cascaded Tuned Amplifiers. It is frequently necessary to incorporate more than one stage of amplification in a given amplifier. Although such a practice provides a higher gain, this higher gain is accompanied by a narrower bandwidth than for the single stage. The situation here is sensibly the same as that which was considered in Sec. 4-7. Analytic expressions for the effect of cascading identical amplifiers are readily possible, following the previous method of analysis.

Consider first n single-tuned stages in cascade. The gain of such an n-stage amplifier becomes, from Eq. (11-15),

$$\left(\frac{K}{K_{res}}\right)^n = \frac{1}{[1 + (2\delta Q_e)^2]^{n/2}} \tag{11-56}$$

To find the corresponding bandwidth, it is noted that

$$[1 + (2\delta Q_e)^2]^{n/2} = \sqrt{2}$$

from which

$$1 + (2\delta Q_e)^2 = 2^{1/n}$$

so that

$$2\delta Q_e = \sqrt{2^{1/n} - 1}$$

But the bandwidth is given by

$$B_{1n} = 2\delta f_0 = \frac{\sqrt{2^{1/n} - 1}}{Q_e/f_0} \tag{11-57}$$

This may be expressed in terms of the bandwidth of the single stage B_1, in the form

$$B_{1n} = B_1\sqrt{2^{1/n} - 1} \tag{11-58}$$

Table 4-1 gives the bandwidth reduction function $\sqrt{2^{1/n} - 1}$. It is seen, for example, that two stages in cascade have a bandwidth that is only 0.64 times that of a single stage. To maintain a given bandwidth, it is accordingly necessary that the Q of the individual stages be decreased as the number of stages is increased.

A corresponding expression is possible for the double-tuned amplifier. For such an n-stage amplifier, with critical coupling $a = 1$ and equal primary and secondary values of Q, the relative gain becomes, from Eq. (11-54),

$$\left(\frac{K}{K_{res}}\right)^n = \frac{1}{(1 + 4\delta^4 Q^4)^{n/2}}$$

It follows from this that

$$\delta Q = \sqrt{\frac{\sqrt[4]{2^{1/n} - 1}}{4}}$$

The bandwidth of the n-stage amplifier then has the form

$$B_{2n} = 2\delta f_0 = 2\sqrt{\frac{\sqrt[4]{2^{1/n} - 1}}{4}}\frac{f_0}{Q}$$

which may be written in terms of one-stage bandwidth as

$$B_{2n} = B_2\sqrt[4]{2^{1/n} - 1} \tag{11-59}$$

The bandwidth reduction factor is tabulated in Table 11-1. For a two-stage double-tuned amplifier with the coils critically coupled, the bandwidth is 0.802 times that of the single-stage amplifier. Note that

TABLE 11-1

THE DOUBLE-TUNED-AMPLIFIER BANDWIDTH REDUCTION FACTOR FOR $a = 1$

n	$\sqrt[4]{2^{1/n} - 1}$
1	1.00
2	0.802
3	0.713
4	0.659
5	0.622
6	0.592
7	0.568
8	0.548

this reduction is considerably less than the corresponding reduction of the two-stage single-tuned amplifier. This arises from the fact that the amplification or selectivity curve of the double-tuned amplifier has steeper sides than that of the single-tuned circuit and with successive stages drops away less rapidly than for the single-tuned case. In particular, an ideal amplifier with a rectangular response curve would show no bandwidth reduction with the addition of successive stages.

11-5. Gain-Bandwidth Product.* It is of interest to tabulate the gain at resonance of the three amplifier circuits that have been studied. These follow:

Single-tuned direct-coupled:	$\mathbf{K}_{res} = -g_m\omega_0 L Q_e$
Single-tuned transformer-coupled:	$\mathbf{K}_{res} = g_m\omega_0 M Q_e$
Double-tuned:	$\mathbf{K}_{res} = -j0.5 g_m Q_1 Q_2\sqrt{R_1 R_2}$

These expressions may be interpreted as showing that the gain in each case has the form

$$\mathbf{K}_{res} = g_m \mathbf{Z} \tag{11-60}$$

where g_m is the transconductance of the tube and Z is the effective impedance of the load. Moreover, the foregoing analyses for these amplifiers show that the bandwidth in each case varies inversely with the effective Q of the tuned circuit. Clearly, therefore, the higher gains are accompanied by a decreasing bandwidth.

The gain-bandwidth product of the single-tuned direct-coupled amplifier is obtained by combining Eq. (11-10) with Eq. (11-16). There results

$$K_{res}B = g_m\omega_0 L Q_e\frac{f_0}{Q_e} = g_m\frac{\omega_0 L}{2\pi}$$

which may be written in the form

$$K_{res}B = \frac{g_m}{2\pi C}$$

In the limit where the capacitance C is due only to interelectrode capacitances, the limiting gain-bandwidth product is

$$K_{res}B = \frac{g_m}{2\pi(C_{in} + C_{out})} \tag{11-61}$$

and is the same value as found in Sec. 4-7 for the RC amplifier.

The gain-bandwidth product of the double-tuned amplifier is found by combining Eq. (11-41) with Eq. (11-55) and is

$$\begin{aligned} K_{res}B &= \frac{g_m}{2} Q_1 Q_2\sqrt{R_1 R_2}\sqrt{2}\,\frac{f_0}{\sqrt{Q_1 Q_2}} \\ &= \frac{1}{\sqrt{2}} g_m\sqrt{Q_1 Q_2 R_1 R_2}\,\frac{\omega_0}{2\pi} \\ &= \frac{g_m}{2\pi}\frac{1}{\sqrt{2}\sqrt{C_{out}C_{in}}} \end{aligned} \tag{11-62}$$

This expression shows that the gain-bandwidth product of the double-tuned amplifier is $\sqrt{2}$ as great as that for the single-tuned circuit. That

is, by splitting the tube input and output capacitances by the use of the double-tuned circuit, there is an increase in the gain-bandwidth product.

11-6. Stagger Tuning.[3] If it is desired to build a wide-band high-gain amplifier, one procedure is to use either single-tuned or double-tuned circuits which have been heavily loaded so as to increase the bandwidth. The gain per stage is correspondingly reduced, by virtue of the constant gain-bandwidth product. The use of a cascaded chain of stages will provide for the desired gain. For example, a particular amplifier comprising nine cascaded single-tuned stages each having a 6 Mc bandwidth has an over-all bandwidth of 1.7 Mc. A nine cascaded chain of double-tuned amplifiers, each also of 6 Mc bandwidth, yields an over-all bandwidth of 3.2 Mc. Generally, for a specified gain and bandwidth the double-tuned cascaded amplifier is preferred, since fewer tubes are often

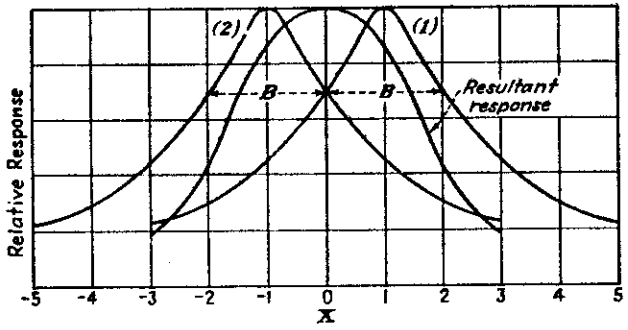


FIG. 11-10. The adjustments of frequency of a stagger-tuned pair.

possible, and also since the pass-band characteristics of the double-tuned cascaded chain are more favorable, falling more rapidly outside the pass band. From practical considerations, the double-tuned stages are more difficult to align, and they also are more sensitive to variations in tube capacitance and coil inductance than the single-tuned circuits.

A means is available for achieving the large bandwidth and other characteristics of double-tuned circuits by using single-tuned circuits. This consists in taking two single-tuned circuits of a certain bandwidth, and displacing, or "staggering," their resonance peaks by an amount equal to their bandwidth. The resultant staggered pair will have a bandwidth that is $\sqrt{2}$ times as great as that of each of the individual single-tuned circuits making up the pair; the over-all selectivity function will be identical in form with that of a single-stage double-tuned system. The general situation is illustrated in Fig. 11-10.

An analytic expression is readily obtained for the over-all characteristics of the stagger-tuned pair. If it is noted that the general selectivity function of the single-tuned direct-coupled circuit is, from Eq. (11-14),

$$\frac{\mathbf{K}}{\mathbf{K}_{res}} = \frac{1}{1 + j2\delta Q_e} = \frac{1}{1 + jx}$$

and the bandwidth between the 3-db points is, from Eq. (11-16),

$$B = 2\delta f_0 = \frac{f_0}{Q_e}$$

then the corresponding selectivity functions of the two circuits are, respectively,

$$\left(\frac{\mathbf{K}}{\mathbf{K}_{res}}\right)_1 = \frac{1}{1 + j(x + 1)}$$

and

$$\left(\frac{\mathbf{K}}{\mathbf{K}_{res}}\right)_2 = \frac{1}{1 + j(x - 1)}$$

By multiplying the selectivity functions together, there results

$$\left(\frac{\mathbf{K}}{\mathbf{K}_{res}}\right)_1 \left(\frac{\mathbf{K}}{\mathbf{K}_{res}}\right)_2 = \frac{1}{2 - x^2 + 2jx}$$

The magnitude of the resulting function is

$$\left(\frac{K}{K_{res}}\right)_1 \left(\frac{K}{K_{res}}\right)_2 = \frac{1}{\sqrt{4 + x^4}} = \frac{1}{2}\frac{1}{\sqrt{1 + 4\delta_0^4 Q^4}} \qquad (11\text{-}63)$$

where δ_0 is the value of δ referred to the new frequency ω_0 and where Q is the value of Q_e for each circuit referred to ω_0. A comparison of this expression with Eq. (11-54) for the double-tuned circuit shows that the forms of the variation are identical.

It is of some interest to compare the gain-bandwidth products of the following: two synchronously tuned stages, a stagger-tuned pair, two synchronously tuned double-tuned stages. The results are found to be, respectively, $0.643K^2B_1$, $0.707K^2B_1$, $1.13K^2B_1$. Thus, not only are the resultant gain-bandwidth products higher for the more complicated coupling systems, but the selectivity is also better.

The advantage of stagger-tuned amplifiers, and the principle may be extended to stagger triples (and to stagger n-uples in general), lies in the fact that simple single-tuned circuits are used throughout. This makes the alignment of the stages relatively easy, especially if stagger triples or higher were to be used, since no interaction exists among tuning elements of the several stages. To attempt a triple-tuned single-stage coupling network proves an almost impossible practical tuning task. A practical disadvantage of the stagger-tuned circuit exists which makes the double-tuned circuit preferable and often almost necessary for 60 Mc and above. The input impedance of an amplifier stage at these high frequencies is low and may be no more than several thousand ohms.

This input loading of the tuned circuit of the previous stage may make it almost impossible to effect stagger tuning for a prescribed over-all bandwidth, whereas this loading affects the double-tuned response characteristics to a lesser degree, with a consequent less stringent restriction on the operation.

As noted, the principle of stagger tuning can be extended, and staggered triples have been used in radar receivers. In this case a centered single-tuned circuit of relative bandwidth 2 and relative resonant gain ½ is combined with two single-tuned circuits, each of relative bandwidth 1 and relative gain 1, staggered so that their resonance peaks are $\pm \sqrt{3}/2$ from the band center. The resulting sensitivity function is of the form

$$\frac{K}{K_{res}} = \frac{1}{\sqrt{1 + x^6}} \tag{11-64}$$

This selectivity function has the same form as that for an optimally flat triple-tuned circuit.

11-7. The Parallel- or Twin-T Circuit.[3] The use of *RLC* resonant networks for achieving band-pass amplifiers with high selectivity has been considered in some detail in the foregoing sections. Such amplifiers are not suitable for use at the low frequencies, since it is difficult to obtain high-Q coils at the low frequencies, and moreover the circuits become rather bulky at these lower frequencies. In fact, a tuned circuit which has a high Q at, say, 30 cps would be extremely difficult to build. Fortunately, a number of *RC* networks possess frequency-selective properties like those of resonant and band-pass filter sections. They have, as a result, found widespread application. The parallel-, or twin-T, circuit is one of this type and will be examined in some detail.

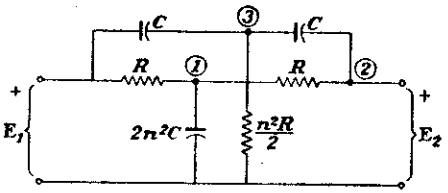


FIG. 11-11. The parallel-T circuit.

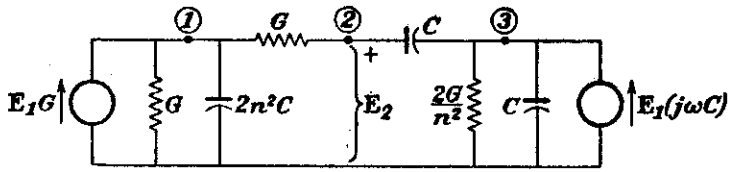


FIG. 11-12. The parallel-T circuit drawn for a junction analysis.

The form of the parallel-T circuit to be analyzed is illustrated in Fig. 11-11. While this is not the most general choice of parameter, it is a form which has received widespread use. To analyze this network, the current-source form of the network is used, and the circuit is rearranged as in Fig. 11-12. The output potential can be written in determinantal form by inspection. It is

$$\mathbf{E}_2 = \frac{\begin{vmatrix} 2(G + j\omega n^2 C) & G\mathbf{E}_1 & 0 \\ -G & 0 & -j\omega C \\ 0 & j\omega C\mathbf{E}_1 & 2\left(\frac{G}{n^2} + j\omega C\right) \end{vmatrix}}{\begin{vmatrix} 2(G + j\omega n^2 C) & -G & 0 \\ -G & G + j\omega C & -j\omega C \\ 0 & -j\omega C & 2\left(\frac{G}{n^2} + j\omega C\right) \end{vmatrix}} \tag{11-65}$$

The expansion of these determinants yields

$$\frac{\mathbf{E}_2}{\mathbf{E}_1} = \frac{1 - \omega^2 n^2 R^2 C^2}{1 - \omega^2 n^2 R^2 C^2 + j2(n^2 + 1)\omega RC} \tag{11-66}$$

Now define the quantity

$$\omega_0 \equiv \frac{1}{nRC} \tag{11-67}$$

and combine with the above. This gives

$$\frac{\mathbf{E}_2}{\mathbf{E}_1} = \frac{1 - (\omega/\omega_0)^2}{1 - \left(\frac{\omega}{\omega_0}\right)^2 + j\,\frac{2(n^2 + 1)}{n}\,\frac{\omega}{\omega_0}}$$

which assumes the form

$$\beta = \frac{\mathbf{E}_2}{\mathbf{E}_1} = \frac{1}{1 + j\,\frac{\omega/\omega_0}{1 - (\omega/\omega_0)^2}\,\frac{2(n^2 + 1)}{n}} \tag{11-68}$$

A plot showing the variation of β as a function of ω/ω_0 for $n = 1$ is given in Fig. 11-13. Note that the ratios $\mathbf{E}_2/\mathbf{E}_1$ are the same for equal values ω/ω_0 and ω_0/ω.

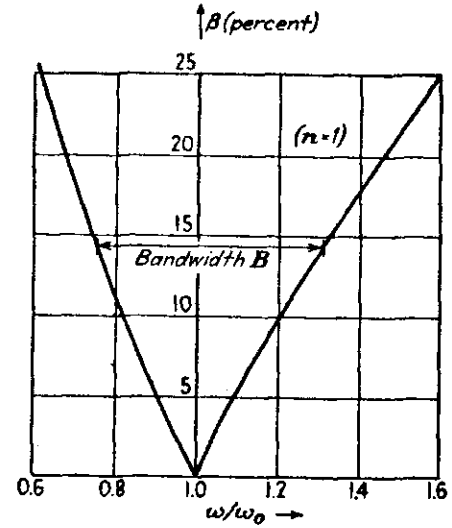


FIG. 11-13. The relation between $\mathbf{E}_2/\mathbf{E}_1$ and ω/ω_0 for the parallel-T circuit.

It should be observed that at resonance $\mathbf{E}_2 = 0$, and the network may be terminated in any impedance $\mathbf{Z}$ across the terminals without any effect. However, a finite terminating $\mathbf{Z}$ will greatly affect the frequency-response characteristics of the network, except at the resonant value, although the general characteristics remain substantially unchanged. For example, if the load is comparable with the network parameters, the off-resonance

response is seriously affected, the selectivity is poor, and the curve is no longer symmetrical about the resonant point.

To find the bandwidth of the network, write

$$\Omega \equiv \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \tag{11-69}$$

Equation (11-68) then becomes

$$\beta = \frac{\mathbf{E}_2}{\mathbf{E}_1} = \frac{1}{1 - j\frac{2}{\Omega}\frac{n^2+1}{n}} \tag{11-70}$$

But the bandwidth is defined by the requirement that the amplitude ratio fall by 3 db over the frequency range. This occurs when

$$\frac{2}{\Omega_b}\frac{n^2+1}{n} = 1$$

or

$$\Omega_b = 2\frac{n^2+1}{n} \tag{11-71}$$

But for $\omega \doteq \omega_0$

$$\Omega = \frac{\omega^2 - \omega_0^2}{\omega\omega_0} \doteq \frac{(\omega+\omega_0)(\omega-\omega_0)}{\omega_0^2} = 2\frac{\Delta\omega}{\omega_0}$$

Hence it is seen that the bandwidth is given by

$$B = 2\Delta\omega \doteq \frac{f_0}{Q} = \frac{\omega_0}{2\pi}\Omega_b \tag{11-72}$$

As already noted, if the network is loaded, the sharpness of the null point will be affected, and the bandwidth or the Q of the network will be correspondingly changed. It is of importance therefore that the input impedance of the network be ascertained. This is readily accomplished by noting from Fig. 11-12 that the total current from source $\mathbf{E}_1$ is made up of two components, that toward junction 1 and that toward junction 3. Evidently

$$\mathbf{I}_1 = \mathbf{E}_1(G + j\omega C) \tag{11-73}$$

which is

$$\mathbf{I}_1 = \mathbf{E}_1\frac{1 + j\omega CR}{R}$$

so that

$$\mathbf{Z}_1 = \frac{\mathbf{E}_1}{\mathbf{I}_1} = \frac{R}{1 + j\omega CR} = \frac{R}{1 + j\frac{n\omega CR}{n}} \tag{11-74}$$

At the resonant frequency $n\omega CR = 1$, and for $n = 1$

$$\mathbf{Z}_1 = \frac{R}{1+j1} = \frac{R}{2}(1 - j1) \tag{11-75}$$

In fact, $\mathbf{Z}_1$ does not vary rapidly in the neighborhood of resonance, and this expression may be used for $\mathbf{Z}_1$ for ordinary calculations. Attention is also called to fact that the network is symmetrical, so that $\mathbf{Z}_2 = \mathbf{Z}_1$.

For the normal circuit parameters used, the output impedance is fairly large. Consequently a load of several megohms might still constitute an appreciable load on the network. This fact will dictate both the location of the grid resistor when such a network is used in an amplifier and also the form of the amplifier circuit. Moreover, the network is inherently a rejection device, and its use in a cascade circuit with the network in either the input or the output of an amplifier will provide a rejection, or "notch," type of device. By combining the network in appropriately chosen feedback circuits, a frequency-selective band-pass amplifier may be achieved. The simplest form is sketched in Fig. 11-14. Loading of

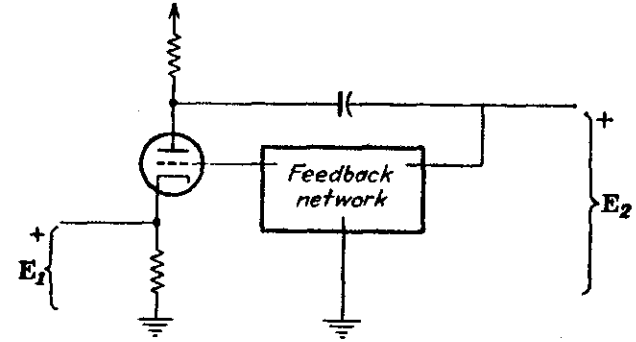


FIG. 11-14. A simple form of parallel-T band-pass amplifier.

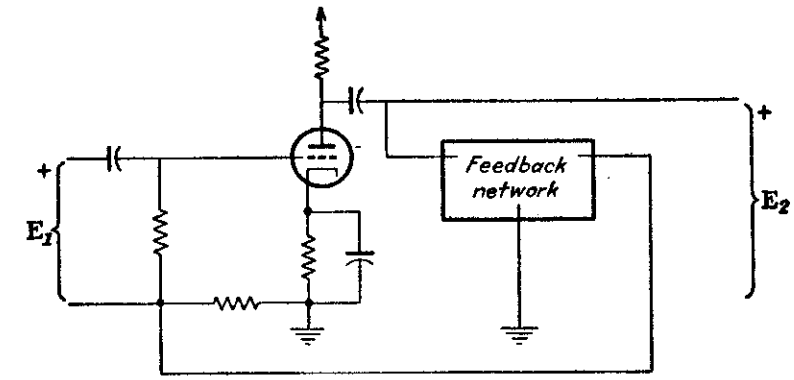
the output of the network is avoided by injecting the signal in the cathode circuit. To avoid loading of the input, more elaborate circuits must be used.

Three circuits incorporating such parallel-T networks in negative feedback amplifiers are given in Fig. 11-15.

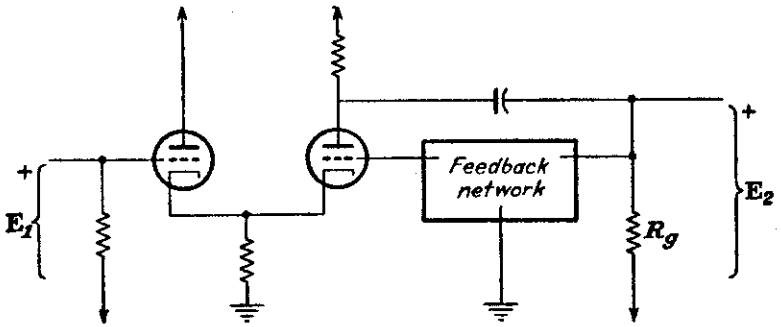
The output response characteristics of such amplifiers are of considerable interest. Observe that in these circuits $\beta = 0$ at the resonant frequency and $\beta \neq 0$ for all other frequencies. Consequently feedback occurs at all frequencies except at the frequency ω_0 at which $\beta = 0$. Thus the circuits have maximum gain at ω_0, and, owing to the feedback, the gain falls at all other frequencies. This means that the circuit attenuates all frequencies except ω_0, so that a "band-pass" amplifier does exist.

If the nominal gain of the amplifier is $\mathbf{K}$, then with feedback the form will be

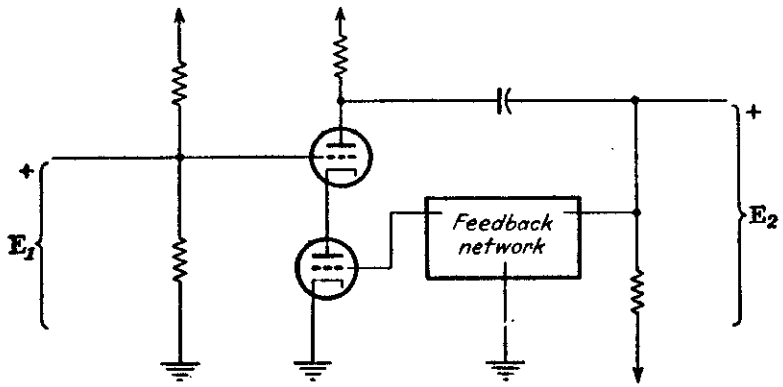
$$\mathbf{K}_f = \frac{\mathbf{K}}{1 - \beta\mathbf{K}} \tag{11-76}$$



(a) Single Tube



(b) Cathode-Coupled



(c) Cascode

FIG. 11-15. Practical parallel-T feedback band-pass amplifiers.

Now combine this expression with Eq. (11-70). The result is

$$K_f = \frac{\mathbf{K}}{1 - \mathbf{K}\dfrac{1}{1 - j\dfrac{2}{\Omega}\dfrac{n^2+1}{n}}}$$

Then

$$K_f = \frac{\mathbf{K}\left(1 - j\dfrac{2}{\Omega}\dfrac{n^2+1}{n}\right)}{1 - \mathbf{K} - j\dfrac{2}{\Omega}\dfrac{n^2+1}{n}} \tag{11-77}$$

from which it follows that

$$\frac{K_f}{K} = \sqrt{\frac{1 + \left(\dfrac{2}{\Omega}\dfrac{n^2+1}{n}\right)^2}{|1 - \mathbf{K}|^2 + \left(\dfrac{2}{\Omega}\dfrac{n^2+1}{n}\right)^2}} \tag{11-78}$$

It is of interest to define the effective Q of this circuit, as a basis of comparison with an RLC circuit. At the half-power points the gain ratio must be $1/\sqrt{2}$, whence

$$\frac{1 + \left(\dfrac{2}{\Omega_b}\dfrac{n^2+1}{n}\right)^2}{|1 - \mathbf{K}|^2 + \left(\dfrac{2}{\Omega_b}\dfrac{n^2+1}{n}\right)^2} = \frac{1}{2}$$

Upon solving for Ω_b from this expression, there results

$$\frac{4\left(\dfrac{n^2+1}{n}\right)^2}{|1 - \mathbf{K}|^2 - 2} = \Omega_b^2 \tag{11-79}$$

Ordinarily $\mathbf{K}$ is negative and greater than 1. Then when $|1 - \mathbf{K}|^2 \gg 2$,

$$\Omega_b \doteq \frac{2(n^2+1)/n}{|1 - \mathbf{K}|}$$

Also, when $-\mathbf{K} \gg 1$, which is the usual case,

$$\Omega_b \doteq \frac{2(n^2+1)/n}{K} \tag{11-80}$$

But by Eq. (11-72)

$$\Omega_b = \frac{1}{Q}$$

Then

$$Q = \frac{K}{2(n^2+1)/n} \tag{11-81}$$

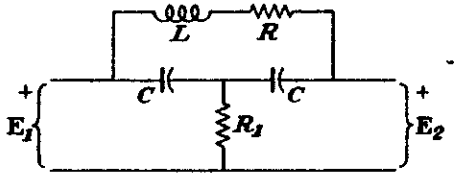
The use of a pentode in a single-tube parallel-T feedback circuit will ordinarily have a nominal gain of 100 or more at audio frequencies. Consequently such an amplifier with a parallel-T loop becomes equivalent to a resonant circuit with a Q of the order of 25 or more, for $n = 1$. A triode amplifier will, because of the lower gain, give a lower effective Q. However, this may be ample for many applications. Of course the cascode circuit of Fig. 11-15*c* has a high gain with triodes and will yield appreciable Q values.

11-8. The Bridged-T and Wien-bridge Circuits. The parallel-T circuit discussed in the foregoing section is only one of a number of different

TABLE 11-2

FREQUENCY-SELECTIVE *RC* NETWORKS

1. Bridged T

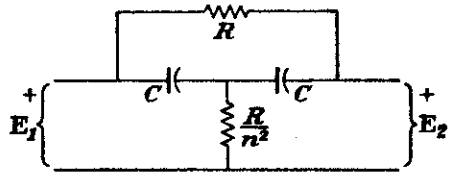


$$\omega_0 = \sqrt{\frac{2}{LC}} = \frac{Q}{2R_1C}, \text{ where } Q = \frac{\omega_0 L}{R}$$

$$\beta = \frac{1}{1 + j\dfrac{\omega/\omega_0}{1 - (\omega/\omega_0)^2}\dfrac{2}{Q_0}} = \frac{1}{1 - j\dfrac{1}{\Omega}\dfrac{2}{Q_0}}$$

$$\Omega_b = \frac{2}{Q_0}$$

2. Bridged T

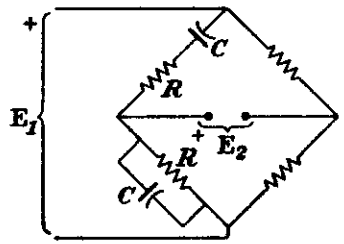


$$\omega_0 = \frac{n}{RC}$$

$$\beta = \frac{1 - j\dfrac{2}{n\Omega}}{1 - j\dfrac{1}{\Omega}\dfrac{n^2 + 2}{n}}$$

$$\Omega_b = \sqrt{\frac{n^2 + 4n - 4}{n^2}} = 1 \text{ for } n = 1$$

3. Wien bridge



$$\omega_0 = \frac{1}{RC}$$

$$\beta = \frac{-1}{3}\frac{1}{1 + j\dfrac{3\omega/\omega_0}{1 - (\omega/\omega_0)^2}} = \frac{-1}{3}\frac{1}{1 - j\dfrac{3}{\Omega}}$$

$$\Omega_b = 3$$

RC frequency-selective networks which may be used to yield band-pass characteristics somewhat like the *RLC* tuned resonant circuits. Among others,[4] the bridged-T (in several forms) and the Wien-bridge networks also have the desired characteristics. The final results are tabulated without proof in Table 11-2 (see Probs. 11-24 to 11-26).